Team _______________

____

Oscillations

Oscillatory motion is motion that repeats itself. An object oscillates if it moves back and forth along a fixed path between two extreme positions. Oscillations are everywhere in the world around you. Examples include the vibration of a guitar string, speaker cone or tuning fork; the swinging of a pendulum, playground swing or grandfather clock; the oscillating air in an organ pipe, the alternating current in an electric circuit, the rotation of a neutron star (pulsar), neutrino oscillations (subatomic particle), the up and down motion of a piston in an engine, the up and down motion of an electron in an antenna, the oscillations of the electron cloud in an atom, the vibration of atoms in a solid (heat), the vibration of molecules in air (sound), the vibration of electric and magnetic fields in space (light).

The Force

The dynamical trademark of all oscillatory motion is that the net force causing the motion is a *restoring force*. If the oscillator is displaced away from equilibrium in any direction, then the net force acts so as to restore the system back to equilibrium.

<u>Definition</u>: A *simple harmonic oscillator* is an oscillating system whose restoring force is a *linear force* – a force F that is proportional to the displacement x :

$$F = -kx \ .$$

The *force constant* k determines the strength of the force and measures the "springiness" or "elasticity" of the system. Slinkys, long pendula, and loose drumheads have small k values. Car coils, short pendula, and tight drumheads have large k values.

The Motion

The motion of *all* mechanical systems is determined by Newton's law of motion $F = ma$. For any simple harmonic oscillator system, characterized by *mass* m and *force constant* k, the equation of motion is $-kx = ma$, or

$$x = -(m/k)\, a \ .$$

This linear relation between position x and acceleration *a* is the kinematic trademark of simple harmonic motion. Note that the proportionality constant (m/k) between x and *a* is the ratio of the *inertia* (m) of the system to the *elasticity* (k) of the system. Also note that m/k has units of time squared: kg/(N/m) = $(seconds)^2$.

The quantity $\sqrt{m/k}$ *sets the scale of time for all simple harmonic motion.*

How does the position x(t) of a simple harmonic oscillator depend on time t? The *motion equation* $x = -(m/k)a$ determines the *motion function* x(t). If you know how to take the derivative of a sine function, then you can easily verify the following important fact:

* *The solution of* $x = -(m/k)\, d^2x/dt^2$ *is* $x(t) = A\sin(2\pi t/T)$ *where* $T = 2\pi\sqrt{m/k}$ *

The *amplitude* A determines the "strength" (maximum displacement) of the oscillation. The <u>*period*</u> T is the time for one oscillation. Note that the sine function $x(t) = A\sin(2\pi t/T)$ is *bounded*: $-A \le x \le A$. Note that the sine function $x(t) = A\sin(2\pi t/T)$ is *periodic* – it repeats itself whenever t increases by the amount T: $x(t+T) = x(t)$. This is the precise definition of "period". The formula for T gives the exact relation between the oscillation time and the system parameter ratio m/k. When you think about it, the dependence of T on m/k makes perfect intuitive sense. Since $T \sim \sqrt{m}$, a "large m system" has a "large T" and therefore "oscillates slowly" – it takes a long time to overcome the large inertia of the heavy mass. In contrast, since $T \sim 1/\sqrt{k}$, a "large k system" has a "small T" and therefore "oscillates rapidly" – it takes a short time for the large restoring force to accelerate the mass back to equilibrium.

<u>Summary: The Essence of Simple Harmonic Motion</u>

IF the force acting on mass m has the *linear* (simple) form: $F = -kx$

THEN the motion of the mass will be *sinusoidal* (harmonic): $x = A\sin(2\pi t/T)$

AND the period T depends solely on the *mass/force* ratio: $T = 2\pi\sqrt{m/k}$

Part I. Pendulum

Pendulum Basics

In lecture, you derived the well-known formula for the period of a simple pendulum:

$$T = 2\pi\sqrt{L/g}\ .$$

The derivation consists of applying Newton's Law $F = ma$ to a mass m suspended from a lightweight (massless) string of length L in a gravitational field of strength g.

The dynamical essence of the derivation is the following. If you pull a pendulum a small horizontal distance x away from its vertical resting (equilibrium) position, then the force F restoring the pendulum back to equilibrium is $F = -(mg/L)\,x$, i.e. F *is proportional to* x !
If you compare the pendulum force function $F = -(mg/L)\,x$ with the general force function $F = -kx$. , then you reach an important conclusion:

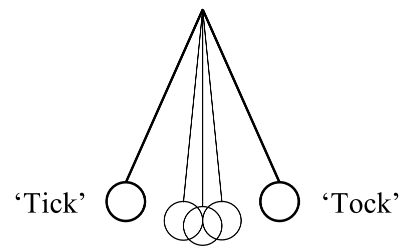
The force constant that characterizes the pendulum system of mass m *and length* L *is* $k = mg/L$.

Once you have the force constant, it is easy to get all the motion properties! To get the period of the pendulum, simply substitute the pendulum constant $k = mg/L$ into the general period formula $T = 2\pi\sqrt{m/k}$. When you do this, note how the *mass m cancels*! You are left with $\underline{T = 2\pi\sqrt{L/g}}$.
This is how the famous pendulum formula is derived.

A. Making a Grandfather Clock

Galileo made the first quantitative observations of pendulum motion by timing the swing of a chandelier hanging from a cathedral ceiling in around 1600. He used his pulse to measure the period. His observations and theory led to the construction of the first pendulum clock by Christiaan Huygens in 1650.

It is now your turn to make a simple "grandfather clock", also known as a *seconds pendulum*. The pendulum in a grandfather clock swings so that the time interval between the 'tick' and the 'tock' is exactly *one second*. Note: the period of a *seconds pendulum* is not equal to one second.



Before you build your clock, calculate the length of a *seconds pendulum* in the space below.

L = ________________ *m* .

Based on your theoretical value of L, construct your clock using a piece of string and a 200-gram brass weight for the pendulum bob. Remember to measure L from the pivot point to the center of mass of the weight. Pull the pendulum $15^o - 20^o$ away from the vertical and release. Measure the period with a stopwatch. Remember the accurate method to measure a period: measure the time for five oscillations and then divide by five to get the period.

T = ________________ *s* .

How well does your clock keep time, i.e. what is the time interval between the 'tick' and the 'tock' ?

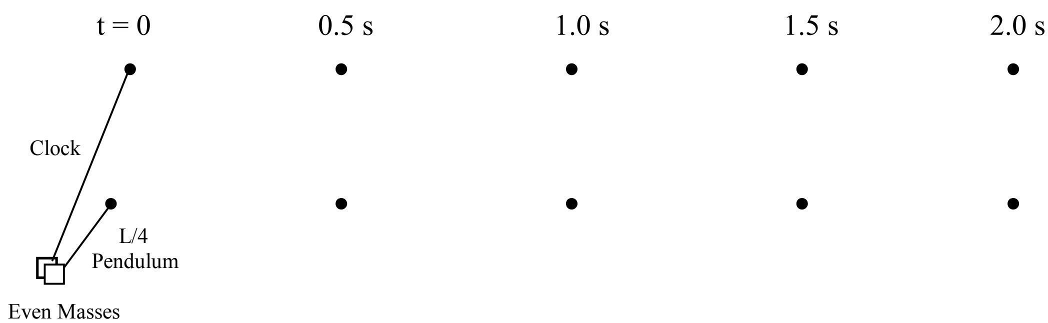
Tick-Tock Time = ________________ *s*.

B. Experimental Test of the Relation T ~√L

Construct a new pendulum that is exactly 1/4 the length of your seconds pendulum. Use a 200-*gram* mass as the pendulum bob. Hang this new pendulum right next to your *seconds pendulum*, but from the lower pivot so that the two masses are at the same horizontal level. Note how the lower pivot bracket allows you to slide the string through the bracket until the 1/4 length is achieved (no need for cutting and tying string).

Measure the period of this new pendulum *without using a stopwatch*. Instead, use your homemade clock that keeps time by 'tick-tocking' in seconds. Pull both pendulum masses a small angle away from the vertical and release at the same time. Carefully observe the parallel motions.

Draw five pictures that show the position of each pendulum at five different times:



By comparing the swing of your seconds pendulum to the swing of the 1/4 length pendulum, how many seconds does it take for the 1/4 length pendulum to complete one back and forth motion?

T (1/4 Length) = ________________ *s* .

Based on your measured values of the period T(L) of a pendulum of length L and the period T(L/4) of a pendulum of length L/4, can you conclude that *period of a pendulum is proportional to the square root of its length*: T ~√L ? Explain carefully. *Hint*: construct ratios.

C. Experimental Test of *"T is independent of m"*

The theoretical formula $T = 2\pi\sqrt{L/g}$ says two amazing things:

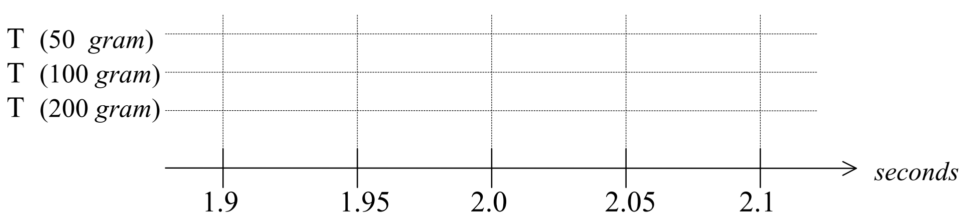
1. The period *does not* depend on the mass of the pendulum.
2. The period *does not* depend on the size of the arc through which the pendulum swings.

Recall that this formula is valid for "small" oscillations or small arcs ($x \ll L$). Let's see if we can experimentally prove that T is insensitive to m.

Use your seconds pendulum. To insure small oscillations, keep the angle of oscillation less than 20^o. Measure the period T for three different masses (m = 50 *gram*, 100 *gram* , 200 *gram*). Remember to measure the time for five oscillations and then divide by five to get the period. For each mass, repeat the period measurement four times. Compute the *average* value of T and the *uncertainty*. Define the uncertainty as the half-width spread in the T values: $(T_{max} - T_{min})/2$.

m (*grams*)	T	T	T	T	*average* T ± *uncertainty*
50					±
100					±
200					±

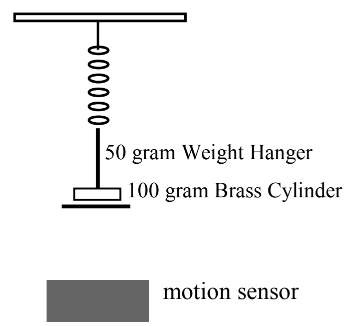
Provide a graphical display of your measured values of T (*average* ± *uncertainty*) on the following *range diagram* , i.e. draw the three horizontal line segments that represent your three T ranges.



Does your range diagram provide experimental proof that the period of a simple pendulum is independent of the mass of the pendulum? Explain.

Part II.   The Mass on Spring Oscillator

Suspend the 50-*gram* weight hanger from the end of the narrow silver spring. Place the 100-*gram* slotted brass cylinder on the hanger platform. Place the motion sensor on the table directly under the oscillator. The distance between the sensor disk and the resting oscillator should be about 25 *cm*. Open Logger Pro file *DisVelAcc*. With the mass in the resting (equilibrium) position, *zero the motion sensor* by clicking *Experiment* on the menu bar and selecting *Zero*. Pull the mass about 10 *cm* away from the equilibrium position (x = 0) and release. Record the oscillations with the motion sensor. If needed, change the scales on your graph so that you can clearly see the sinusoidal variations.



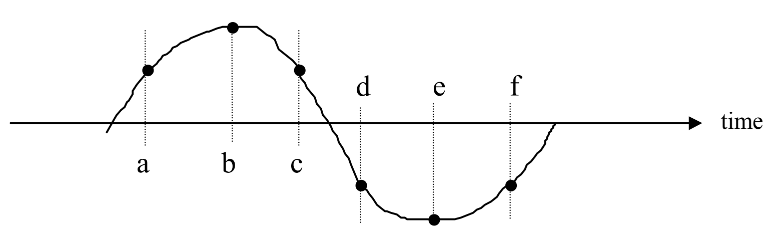
A.   x(t) and *a*(t) are "In Sync".   Finding T from x(t) / *a*(t).

The kinematic trademark of simple harmonic motion is that the position x(t) and the acceleration *a*(t) are proportional to each other, and opposite in direction, for *all time* t:

$$a(t) \;=\; -\,\omega^2\, x(t)\ .$$

The proportionality constant between *a*(t) and x(t) is the square of the angular frequency $\omega = 2\pi/T$ of the system. When x is zero, *a* is zero. When x is maximum (minimum), *a* is minimum (maximum).   Note that the velocity is completely out of sync (out of phase) with x and *a*. When x is zero, v is max.

Examine your graphs to be a first-hand witness to this *"in-sync dance"* between x and *a*. Do your x(t) and *a*(t) curves satisfy the motion equation $a(t) = -\,\omega^2 x(t)$?   Change the time scale to go from 0 to 3 *s*.   Use the *Examine Icon* [x = ?] to find the values of x(t) and *a*(t) at the six times t = a, b, c, d, e, f shown below. Any six points that are *approximately* equally-spaced within one period of the sine curve are perfectly fine.



Record your measured values of x and *a* along with the ratio x/*a* in the table:

t	x(t) (*m*)	*a*(t) (m/s^2)	x(t) / *a*(t) (s^2)
a			
b			
c			
d			
e			
f			

According to the theory, x(t) and *a*(t) both *vary* in time, but the ratio x(t)/*a*(t) is *independent* of t ! Your experimental data should support this theoretical fact. Based on your data, what is the constant value of x(t)/*a*(t) that characterizes your mass-spring system?

x(t) / *a*(t) = ________________ ± ______________ s^2 .

Calculate the period T of your oscillator from your measured value of x(t)/*a*(t). Remember, $a(t) = -\omega^2 x(t)$ and $\omega = 2\pi/T$. Show your calculation.

T = _______________ *s* .

B. Finding T from m/k

The mass of your mass-spring system is <u>m = 0.150 kg</u> . The force constant k is equal to the spring constant of the spring in your system. Measure the value of k. You have measured spring constants before. Hang a weight (F) from the spring and measure the stretch (x). Repeat for several weights. Plot F versus x and find the best-fit slope. Hand in the graph showing the best-fit line and the “slope box”. Report your k value. Find the *m-over-k* value of your oscillator.

k = _______________ *N/m* .   m/k = _______________ s^2 .

Calculate the period of your oscillator from your measured value of m/k. Show your calculation.

T = _______________ *s* .

C. Summary

You have measured the period of a mass-spring oscillator using two different methods. As a third method, measure the period directly with a stopwatch. Summarize your results by listing all three of your T-values below.

<u>*Motion*</u> Method. Measure x(t) and *a*(t), compute x(t)/*a*(t) : T = ______________ *s* .

<u>*Force*</u> Method. Measure m and k, compute m/k : T = ______________ *s* .

<u>*Timing*</u> Method. Measure time directly with clock : T = ______________ *s* .

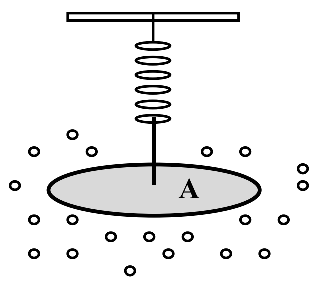
Part III. Damped Oscillator – A Class Research Project

So far, we have neglected the effect of friction on the motion. In physics, an oscillator subject to friction is called a "damped oscillator". Friction causes the motion to decay. The amplitude of the oscillations decreases as time goes on. The height of the sine curve gets smaller and smaller. In this experiment, you will study the effect of air friction on the motion of your mass-spring oscillator.

Any object moving through the air experiences a 'viscous drag force' due to the myriad of impacts between the object and the air molecules. As a result of the air-object collisions, the object slows down and the air molecules speed up (the air gets hotter). *Mechanical energy* of the object is transformed into *thermal energy* of the air.

The System

Each team will attach a cardboard surface of *area* A to their oscillator. Unscrew the bottom disk from the weight hanger rod. Put the rod through the hole in the board. Screw the disk back onto the rod. Place the 100 *gram* slotted brass cylinder on top of the board. You now have a damped harmonic oscillator:



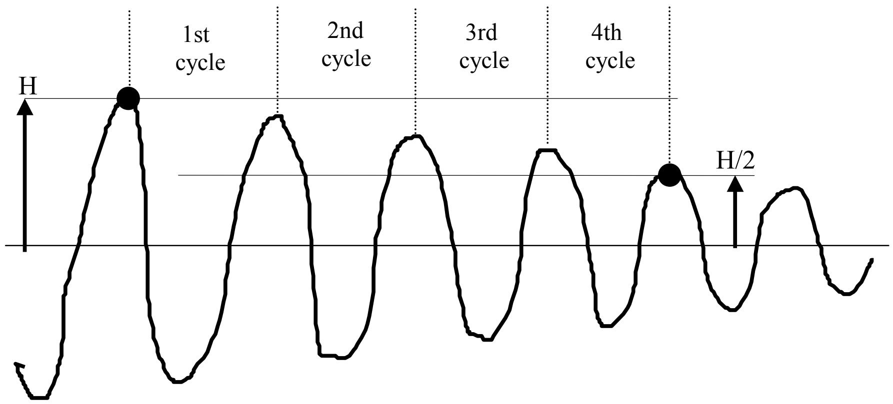
As your oscillator moves up and down, imagine the cardboard surface pushing through the *"sea of air"*. The numerous collisions (about 10^{22} per second!) between the surface and the air molecules (tiny "balls" of nitrogen and oxygen) are the mechanical source of the air friction.

Each team will get a different surface *area* A to study the effect of air friction on the decay of the motion. The goal of the class is to find how the *amount of decay* depends on the *magnitude of the area*. This is the kind of research conducted by scientists and engineers who study the aerodynamic design of cars, planes and rockets, and the hydrodynamic design of ships and submarines.

Half Life

How are you going to quantify the "amount of decay"? You will measure the ***half-life*** of the oscillator. Scientists use the concept of half-life to quantify the "*lifetime*" of anything that decays, such as a pendulum with friction, a circuit with resistance, an atom in an excited state, an unstable (radioactive) nucleus, and a bacteria colony.

The half-life is the time it takes for the amplitude of the motion to decay to one half of some initial value. The following position-time curve of a damped oscillator illustrates the concept of a half-life:



Note that the amplitude (height) decreases from H to H/2 in 4 cycles (periods). The half-life is therefore 4T, where T is the period. It is easier to measure the half-life simply by the dimensionless number:

$N_{1/2} \equiv$ Number of cycles it takes for the amplitude to decrease by a factor of 1/2 .

For the above example, you would report the half-life as $N_{1/2} = 4$. Note that a small value of $N_{1/2}$ corresponds to a large amount of decay and a short lifetime. If there is no damping whatsoever, then $N_{1/2} = \infty$. An undamped (friction free) oscillator "lives" forever.

The Quest

Now that there are two experimental variables (A , $N_{1/2}$) that can be measured, the interesting question is: *How are* A *and* $N_{1/2}$ *related*? One thing is certain. More surface area (big A) means larger damping (small $N_{1/2}$). The half-life $N_{1/2}$ must decrease as the area A increases.

Class Research Goal: Find the function $N_{1/2}(A)$ that describes how $N_{1/2}$ depend on A.

The Experiment

Place the motion sensor directly under your oscillator. The distance between the sensor and the resting oscillator should be about 25 *cm*. Open Logger Pro file *Moving Along*. Change the graph scales so that t goes from 0.0 *s* to 30.0 *s* and x goes from −0.15 *m* to 0.15 *m*. With the oscillator at rest, *zero the motion sensor* [click on *Experiment* and select *Zero*] . Pull the mass away from the equilibrium position (x = 0) about 10 *cm* and release. Record the motion x(t).
PRINT the graph – the *worldline* of a *damped harmonic oscillator*.

Find the half-life $N_{1/2}$ of your oscillator. Start with the first maximum on your x-t curve and find the height H (amplitude) of this maximum. Examine your decaying sine curve to find the maximum point in the future that has the height H/2. Clearly mark these two maximum points on your printed graph. Label the heights of the two horizontal lines that go through these points. Count the number of cycles (periods) between the H point and the H/2 point. Record the values of the surface area and the half-life of your damped harmonic oscillator.

A = ______________ m^2 . $N_{1/2}$ = _______________ .

Write your results in the class data table on the chalkboard. Copy the class data table here:

A (m^2)						
$N_{1/2}$						

After all teams have entered their data points (A , $N_{1/2}$) into the table, plot $N_{1/2}$ (y axis) versus A (x axis). Delete the connecting lines between the points on your graph. Find the best-fit curve through the data points. Try a polynomial fit. First try *quadratic* (polynomial of degree 2). If the fit looks good, then keep it! If you are not satisfied, then try *cubic* (polynomial of degree 3). PRINT your graph showing the best-fit curve. Remember that "y as a function of x" in the curve-fit program is "$N_{1/2}$ as a function of A" for your real-world system. Report the $N_{1/2}$ function for your damped oscillator:

$N_{1/2}$ (A) = ____ .

Note: You will not find any theoretical formula in your textbook that you can use to check your experimental result. Your Physics 150 team is the first in the world to discover the $N_{1/2}$ formula!

Part IV. Coupled Oscillators and Normal Modes

If you couple together two or more oscillators, then the resulting motion gets very interesting. Many systems in nature (mechanical, electrical, atomic) are coupled oscillators. The concept of a "normal mode" simplifies the analysis of the motion of coupled oscillators. It allows you to understand complicated non-sinusoidal vibrational motion in terms of a sum or "superposition" of simple sinusoidal (harmonic) vibrations. The *whole motion* is broken down into a set of more *manageable parts*. These simple parts are the "normal modes".

What exactly is a normal mode?

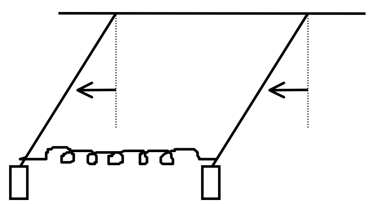
> *A normal mode is a pattern of vibration in a system of particles in which every particle vibrates sinusoidally with the same frequency* f *(the amplitude* A *can be different). Mathematical trademark: The displacement of the n^{th} particle is* $A_n \sin\pi f t$.

Normal Modes A and B

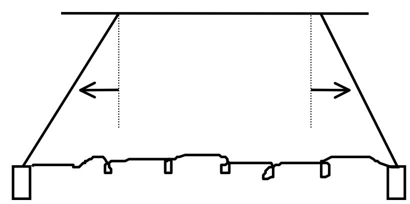
Examine the coupled pendula system on display in the lab. Note that the spring serves as the coupling mechanism. This system may look simple, but it exhibits a rich variety of motions.

There are two normal modes: mode A and mode B. To "excite" mode A, pull both masses away from the vertical (equilibrium) in the *same direction* and release at the *same time*. To "excite" mode B, pull both masses away from the vertical (equilibrium) in *opposite directions* and release at the *same time*. Note that in each normal mode, both masses oscillate with the same frequency. Measure the period T and compute the frequency f of each mode:

Mode A



Mode B

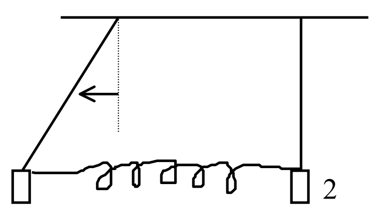


T_A = _______________ *s* . T_B = _______________ *s* .

f_A = ________________ *Hz* . f_B = _______________ *Hz* .

Energy Exchange Motion = Mode A + Mode B

Pull mass 1 away from the vertical (do not touch mass 2) and release.



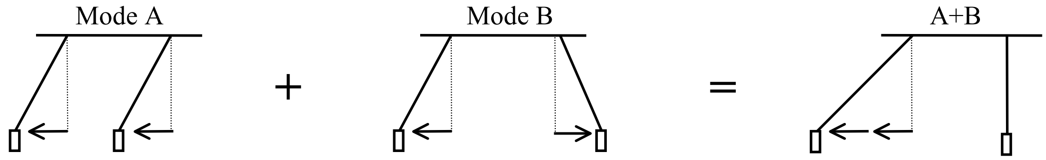
Observe this "energy exchange motion". Note that the total amount of energy in the system remains constant, but it is constantly being passed between the two pendula. Pendulum 2 speeds up at the expense of pendulum 1 slowing down, and vice versa. The energy "sloshes" back and forth between the two parts of the system. The spring provides the transfer mechanism. If there were no friction, then this exchange would go on forever! This simple system models the complex energy transfers that occur in nature.

Measure the period of the energy exchange, i.e. the time it takes for 2, which starts with zero energy, to gain all the energy from 1, and then give it all back to 1. In other words, find the time it takes for pendulum 2 to complete the cycle *rest* → *moving* → *rest*. Since the energy exchange motion is a combination of the A and B modes, we will label this motion "AB". Report your measured values of the period T_{AB} and frequency f_{AB} of this energy exchange (AB) motion:

T_{AB} = _______________ *s* . f_{AB} = _______________ *Hz* .

Relation Between f_A, f_B, f_{AB}

Why is the energy exchange motion a superposition of mode A and mode B? The following picture illustrates how the A and B displacements add to give the energy-exchange displacement:



According to the theory (normal mode analysis), the frequency of the energy-exchange motion (AB) is related in a simple way to the frequency of the constituent modes (A and B):

$$f_{AB} = f_B - f_A .$$

Do your experimental values of f_{AB}, f_B, and f_A obey this theoretical relation? What is the percent difference between $f_B - f_A$ and f_{AB}?

Music, Atoms, & Photons

There exists many systems and processes in nature where three frequencies are related via the "normal mode rule" $f_{AB} = f_B - f_A$. Most notably are *"beats"* in music and *"photons"* in quantum mechanics.

When two musical notes (sound waves) of slightly different frequency f_A and f_B are played together (superimposed), the result is a sound wave that pulsates at the frequency $f_{AB} = f_B - f_A$. When an electron in an atom makes a *quantum jump* from a high-energy state B, where it "orbits" at frequency f_B, to a low-energy state A where it "orbits" at frequency f_A, the atom emits a light wave of frequency $f_{AB} = f_B - f_A$. During the quantum jump, the electron is in a superposition of quantum states (modes): state A + state B.

In music, the superposition of sound waves results in *"beats"*. In quantum mechanics, the superposition of quantum waves results in a *"photon"*. One of the great triumphs of quantum mechanics was to discover the precise connection between the *orbital frequencies* (f_A, f_B) of the electron in an atom and the *optical frequency* (f_{AB}) of the emitted light. The coupled pendulum system that you studied in this lab provides insight into how two tuning forks make sound (beats) and how two atomic states make light (photons)!