Electric Fields II

Introduction
This week's lab has two parts. In the first part, you will continue using the EM Field computer program to investigate the properties of electric fields. This week you will explore the electric field produced by a line of charges, gain experience with the concept of electric flux and verify Gauss’ Law. In the second part, you will create and measure the properties of a real two-dimensional electric field.

Part I: Exploring a Computer Simulated Electric Field

1. The field due to a line charge

a. What is a line charge?

Introduction
If you charge a conducting rod the charge will spread over the entire rod. If there are no other nearby charges, the charge will spread uniformly over the surface of the rod. Such a charge distribution can be characterized by a single number, the linear charge density, \( \lambda \). It quantifies the amount of charge per unit length along the rod. We will simulate such a charge distribution in EM Field by placing point charges at equal distances along a line.

Procedure

- Select File>Get charges or currents from file from the menu bar. Double-click on qline.emf. Observe that there is one positive unit of charge at each grid point. The linear charge density, \( \lambda \), could be given as +1 charge unit per grid unit. In SI units, one unit of charge per unit of length would be 1 Coulomb per meter, or +1C/m.

- There are two ways to change the charge density. The first would be to increase the amount of charge on each grid point from 1 to 2 units, thus giving 2 charge units per grid units, or 2 C/m in SI units. A second way is illustrated with the file qline2.emf. Open this file and describe the method by which the linear charge density was increased. In addition, state a numerical value for the linear charge density.
b. Electric field strength versus distance from a line charge

Introduction
You know that the magnitude of the electric field decreases with the square of the distance from a point charge; symbolically, \( E \propto 1/r^2 \). Does this pattern hold true as you move away from a line of charge? In this section you will perform a quantitative investigation of the field magnitude’s dependence on distance form a uniform line of charge.

Procedure
- For both \( qline.emf \) and \( qline2.emf \) drop field vectors 1, 2 and 4 grid units from the line of charge. Measure the lengths of each field vector with a ruler and record your results in Table 1.

<table>
<thead>
<tr>
<th></th>
<th>qline.emf</th>
<th></th>
<th>qline2.emf</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (grid units)</td>
<td>Field Vector Length (cm)</td>
<td>Distance (grid units)</td>
<td>Field Vector Length (cm)</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1

- Use the program *Graphical Analysis* to determine the power law behavior of each fields’ dependence on distance by fitting curves to \( E \) versus \( r \) using the functional form \( y = A*x^B \). What is \( B \) to two significant digits for each? (You do not have to print the graphs.)

Based on this result, if distance from the rod is doubled, by what factor will the field decrease?

Comparing your results for \( qline.emf \) and \( qline2.emf \), how does the power law behavior depend on linear charge density? Justify your answer.

- *EM Field* allows you to view a line charge from one end. Select *Sources>*2D charged rod from the menu bar. Place a rod near the bottom of the screen. Drop a few field vectors to find out how the electric field depends on the distance and direction from the rod. Are your results consistent with the \( qline \) results from above? Justify your answer.
Conclusions

Based on your observations and analysis does the field strength as you move away from a line charge decrease more rapidly, less rapidly or just as rapidly as when moving away from a point charge? Justify your answer.

2. Gauss’ Law

Introduction

Gauss' Law is one of the most powerful, yet mystifying, statements in physics. Its mathematical form is,

\[ \frac{1}{4\pi} \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0} \]

In words, Gauss' law states that if you enclose a charge in a closed mathematical surface, then the electric flux integrated over that surface is proportional to the charge enclosed by that surface. The electric flux is defined as the product of the component of the electric field perpendicular to the surface times the area of the surface. What is the meaning of flux and Gauss' law? EM Field will help you get a feeling for the meaning of both. For a brief explanation of how EM Field shows the flux through a surface, select Option>How we display Gauss' law from the menu bar. Read the text, clicking on the screen for each new graph.

a. Flux from a single charged rod

Procedure

- Select a 2D charged rod with charge density +3 and place it in the middle of the screen. (You should still be in the “end-on” mode for viewing the charged rod.)

- Follow these steps to draw a Gaussian surface around the rod. First select Field and Potential>Flux and Gauss’ law from the menu bar. Then use your mouse to draw a circle with a radius one or two grid units around the rod. When the cursor is near the starting point you can release the button and let the program complete the circle for you. Remember that this circle is actually the end view of a long cylinder enclosing the rod. Note that the electric field vector was always (approximately) perpendicular to the surface. The gray bars outside the circle represent the flux through each strip of the cylinder. "Q = 3" means that the value of the charge enclosed in the cylinder is three units.

- Draw a second circle with a much larger radius around the rod. The program again says "Q = 3," which means the total flux was the same as in the previous case, even though the electric field was much smaller at the surface.

How can the total flux be the same through both of these surfaces? (You must not answer, even though true, “Because the same amount of charge was enclosed.” This is not an explanation. You should consider the result, Q = 3, as just that, a result, the endpoint of a calculation by the computer of the net flux through the Gaussian surface, which by Gauss’ Law is proportional to the net charge enclosed. In expanded form, you are being asked, “Even though the electric field at each Gaussian surface is very different, one much smaller than the other, how is it that the integration to determine the net flux results in the same answer, indicating that the charge enclosed in each case, is indeed, the same?” To explain by saying, “Because
the charge enclosed is the same”, is to explain the end result with the result itself; and therefore, incorrect. Ask your instructor if you are confused. In many of the following questions you will have to answer in the same fashion, having to avoid referring to the amount of charge enclosed as an explanation.)

- Select Sources>Add more sources from the menu bar. Place a rod with charge density -3 in the center of the two circles and remove the +3 rod from the screen. What is different about the flux and how is it represented by the program?

- Remove the -3 rod from the screen and place a +3 rod in the center of the circles again. Select Display>Unconstrain so that you can place the rod anywhere on the screen. Now move it near the surface, but still inside the inner circle. The flux, and field, near the charge is much larger, but the total flux is still given by "Q = 3." Explain how this can be. (Again, you must not answer, “Because the charge enclosed is still the same.”)

- Finally, move the +3 rod outside the inner circle. The program says that the enclosed charge, and thus the total flux through the inner surface, is now zero. Explain how that comes about. (Again, do not answer, “Because the charge enclosed is zero.”)
Part II: Exploring a Real 2-dimensional Electric Field

Introduction

You were first introduced to the field concept with the gravitational field. Gravity, you remember, is not a force that acts when two objects touch. It is a force that seems to act over a distance. Physicists have never been happy with the idea of “action-at-a-distance.” Rather, they prefer to think about a gravitational field that is created by one object, like the sun. The field exists at the location of the second object, like Earth, and acts on it there. Thus the force is a local, not a distant force. In the same way, physicists have found it useful to describe the electric interaction in terms of an electric field created by one charge acting on a second charge at the location of that charge.

Defining the electric field

In principle, a field can be measured by measuring the force on an object located in that field. The force is then divided by the object’s charge, \( \vec{F} = q \vec{E} \), to obtain the electric field, or mass, \( \vec{g} = \vec{F}/m \), to obtain the gravitational field. The units of the electric field are Newtons/Coulomb.

Electric field lines

The electric field can be visualized by means of field lines. An electric field line is a line which is at every point tangential to the electric field at that point. In other words, a positive charge, released from rest, would start accelerating along the field line. (Note, however, if the field line curves, the charge will not simply continue following along the field line.)

Electric potential difference

A direct measurement of a field is very difficult. We will use an indirect way of measuring the electric field that will involve the measurement of electric potential difference.

The electric potential difference is defined as the work done on a unit positive charge in moving the charge from one point to another in an electric field. The units of potential are work per charge, which, in SI units, is Joules/Coulomb. One Joule/Coulomb is defined as a volt.

As is shown in the textbook, the electric field, in one dimension, is the derivative of the potential. For a three-dimensional field, we must use the three dimensional rate of change of the potential. Thus, in component form,

\[
E_x = -\frac{dV}{dx} \quad E_y = \frac{dV}{dy} \quad E_z = -\frac{dV}{dz}
\]

From these expressions it should be clear that another unit for electric field is potential per unit distance which, in SI units, is volts/meter. (You can check to see that V/m and N/C are equivalent units.)

Because \( \vec{E} \) is related to a spatial derivative of \( V \), we can use a difference method of measuring \( \vec{E} \). For points \( a \) and \( b \) close together we approximate the derivative by a difference equation,

\[
E_x = -\frac{dV}{dx} \approx -\frac{\Delta V}{\Delta x} = -\frac{V_b - V_a}{x_b - x_a}
\]

The potential difference, \( V_b - V_a \), can be measured with a potential-difference meter, commonly called a voltmeter. The quantity \( x_b - x_a \) is a distance, and can be measured with a ruler. From these two measurements we can calculate \( E_x \). We can do similar measurements for the other components, \( E_y \) and \( E_z \).
The negative sign means that the electric field component along any axis points opposite to the direction in which the potential increases. Thus, the electric field vector at any point in space always points opposite to the direction of the greatest rate of change of potential, and has magnitude equal to this rate of change.

It is also useful to define the concept of equipotential lines and surfaces. A point charge can be moved without doing any work along an equipotential line (in 2 dimensions) or along an equipotential surface (in 3 dimensions). If no work is done, then the potential must be the same everywhere. Clearly there must be zero electric force in the direction of motion along an equipotential. This is all you need to know about electric fields and potentials to begin the experiment. You will discover some of their properties from your observations.

The experiment

In this experiment, we will set up a two dimensional electric field in a thin sheet of slightly conducting material (called teledeltos paper) by applying a potential difference between two electrodes. Everything we have said so far holds true in this configuration. The potential difference means there is an electric field, and the force due to the field causes a flow of charge (current) in the sheet. We can be sure that the direction of the electric field is parallel to the faces of the sheet because, if it were not, charge would flow to the faces and build up until the field became parallel. Potential differences are measured by a voltmeter. In this experiment, we can use a type of meter that does not require very much current to operate. By using such a meter, we can be sure that the field patterns are not distorted by the presence of the meter.

1. Equipotential lines

Procedure

- Place a sheet of the conducting teledeltos paper on the mapping board that has two bolts through it. Use a sharp point to create clean punctures in the teledeltos paper, rather than simply forcing it onto the bolts. The washer should be resting on the top face of the paper, not the bottom face. Tighten the wingbolts to hold the washers firmly against the teledeltos paper. The bolts/washers provide two point-like electrodes in the plane of the paper.

- Connect the power supply, digital multimeter (DMM) and potential probe as shown in Figure 1. Set the DMM to read dc volts. The potential probe is the one with only one point. (The dual-probe consisting of two single-probes fastened together will be used in a later experiment.)

Figure 1: Wiring diagram for measuring equipotential lines
• Now touch the probe gently to the left-hand electrode. Adjust the power supply to read 12V on the multimeter.

• You are now ready to look for equipotential lines. Gently probe for a point where the multimeter reads 10V. Mark this point with a pencil. Now find enough other points at 10V to be able to draw a smooth equipotential line connecting the points. You can probably make a closed curve.

• Repeat this procedure, but this time mark off the equipotential line at the 8V potential. Make sure you measure enough points so that you can draw a smooth curve through the points. Can you make a closed curve this time?

• Repeat this procedure for equipotentials at 6, 4, and 2 volts. Each time, make sure you mark enough points so that you have an accurate indication of the shape and spacing of the equipotential lines.

• Label the equipotential lines with their potentials.

2. Field lines

Introduction

The field probe consists of two points spaced about 1 cm apart. The two wires of the electric field probe will be connected to the DMM (one wire to the black (com) terminal, one wire to the red ("VΩ") terminal). Connected in this manner, the field probe measures the potential difference between the two points.

You can find the direction of the field by first touching the black probe to the paper as before. Next, without removing the black probe, gently touch the red probe to the paper and read the potential difference. Keeping the black probe fixed in place, slowly pivot the red probe, trying different directions, until you find the orientation that produces the maximum negative potential difference.

The DMM reads the potential difference, \( V_{\text{red}} - V_{\text{black}} \), between the two points, 1 cm apart, that are in contact with the probes. The magnitude of the electric field, in N/C or V/m, midway between the two points, is the magnitude of the potential difference, in volts, divided by 0.01 m. The direction of the electric field at the midpoint is on the line from the black probe (high potential point) to the red probe (low potential point).

Prediction

First, make a prediction for the very quantity that you will measure. You will be measuring the electric field on a line segment that starts close to the high potential (12V) electrode and ends close to the low potential (0V) electrode. The distance between electrodes is about 23 cm and you will be measuring the electric field every 1 cm along that length. How do you think the electric field will change in magnitude from one end to the other? Make a qualitative sketch of your prediction on the graph below. Remember, this is a graph of electric field, not electric potential.
How does the field change as you move from left to right along this line?

Check your thinking

Before making the measurements check your thinking. To do so, first analyze some of the data you collected in the previous section when you mapped out equipotential lines. Along the straight line segment connecting the high potential electrode to the low potential electrode record the distances from the center of the 12 V electrode at which the potential is 10 V, 8 V, 6 V, 4 V and 2 V.

<table>
<thead>
<tr>
<th>Position</th>
<th>Potential</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 V</td>
<td></td>
</tr>
<tr>
<td>8 V</td>
<td></td>
</tr>
<tr>
<td>6 V</td>
<td></td>
</tr>
<tr>
<td>4 V</td>
<td></td>
</tr>
<tr>
<td>2 V</td>
<td></td>
</tr>
</tbody>
</table>
Now graph this data with position on the horizontal axis and potential on the vertical axis.

![Graph of potential vs. position]

Draw a smooth curve that “best fits” the data. This does not mean draw straight-lines-to-connect-the-dots. It is a smooth curve. Nor does the smooth curve have to pass through every data point. This is an attempt to interpret the pattern these few data points are conveying, an attempt to extrapolate from the data the functional form of potential vs. position.

Due to the symmetry of the system the electric field is entirely in the $x$-direction at all points along the line connecting the electrodes. Therefore, the relationship between the magnitude of the electric field and the electric potential on this line is:

$$E_{\text{along connecting line}} = -\frac{\partial V}{\partial x}$$

This is telling us that to determine how the electric field changes along the line that connects the electrodes we can graph how rapidly the potential changes with respect to position along this line (with an overall negative sign). Do so! On the plot below sketch a graph of the negative of the derivative of the “best-fit” curve from the previous graph of potential vs. position.

![Graph of electric field vs. position]

You should consider this plot to be a “well-thought-out” prediction of what you are about to measure. Does it match the prediction you made earlier? Comment on any ways in which your earlier prediction was different.
**Procedure**

- Connect the field probe to the meter as described above and illustrated below in *Figure 2*. Have the DMM set to measure dc volts.

![Figure 2: Wiring diagram for field line mapping](image)

- Starting with the black probe a few millimeters directly to the right of the left (12 V) electrode, pivot the red probe until you find the direction of maximum negative potential difference. Make pencil marks at both probe points. Record this maximum potential difference directly on the teledeltos paper near the probes. (Don’t forget to note the units.)

- Move the probe so that the black probe is where the red probe was, and repeat this procedure. Again, pivot the red probe until you find the direction of maximum negative potential difference.

- Repeat this procedure all the way out to the second electrode.

- Create a field line by connecting the dots. Place arrows on the field line in the direction of the field.

- Repeat this entire procedure to create one other field line. Again, start at the left electrode, but this time, start off in a different direction. If your first field line happened to be the one that went straight across from one electrode to the other, make sure you start in a direction that gives a curved field line this time. If your first field line was nicely curved, this time head straight across from one electrode to the other. (Make sure you are again recording the potential difference on the teledeltos paper.)

- Choose the line that is closest to a straight line from one electrode to the other. Fill *Table 2* with magnitude of potential difference, $|\Delta V|$, and position, $x$, data as you move from the left to right electrode. Transfer potential difference data from the teledeltos paper. The position, $x$, is the distance from the left-hand (12 V) electrode to the midpoint of the two points marking the contact points of the probes. These you can measure with a ruler.

- Fill the table’s third column by calculating $E$ for each point. Because $\Delta x = 1 \text{ cm} = 0.01 \text{ m}$, $E$ is just $|\Delta V|/0.01\text{m}$.

- Use *Graphical Analysis* to create a graph of $E$ versus $x$. Format the graph with an appropriate title and axes labels. Print it and attach it to your lab report along with your teledeltos paper.
| #  | $|\Delta V|$(V) | $\Delta x$(m) | $E$(V/m) | $x$(m) |
|----|-----------|-------------|----------|--------|
| 1  |           |             |          |        |
| 2  |           |             |          |        |
| 3  |           |             |          |        |
| 4  |           |             |          |        |
| 5  |           |             |          |        |
| 6  |           |             |          |        |
| 7  |           |             |          |        |
| 8  |           |             |          |        |
| 9  |           |             |          |        |
| 10 |           |             |          |        |
| 11 |           |             |          |        |
| 12 |           |             |          |        |
| 13 |           |             |          |        |
| 14 |           |             |          |        |
| 15 |           |             |          |        |
| 16 |           |             |          |        |
| 17 |           |             |          |        |
| 18 |           |             |          |        |
| 19 |           |             |          |        |
| 20 |           |             |          |        |
| 21 |           |             |          |        |

*Table 2*

- How does the actual electric field vs. position along the line connecting the electrodes compare with your initial prediction and with the second “well-thought-out” prediction?
• Based on your graph, is the electric field constant between the two electrodes? If not, is there a region in which it is very nearly constant? Describe the region.

• In what region, or regions, of the line between the two electrodes is the electric field strongest? Is the electric field zero anywhere along the line between the two electrodes? If so, where?

• Draw the field and potential lines between the point and straight-line electrode shown below. Hint: you need only consider the data you have taken and the symmetry of the electrodes.

• Explain, in your own words, how the direction of the field at a point on the Teledeltos paper is determined with the field probe method.

• Explain, in your own words, how the magnitude of the field at a point on the Teledeltos paper is determined with the field probe method.