Name ____ Date ____________ Time to Complete ____h ____m

Partner ____ Course/ Section ______/_______ Grade ___________

Electric Fields II

Introduction

This week's lab has two parts. In the first part, you will continue using the *EM Field* computer program to investigate the properties of electric fields. This week you will explore the electric field produced by a *line* of charges, gain experience with the concept of electric flux and verify Gauss' Law. In the second part, you will create and map both field lines and equipotential lines of a real two-dimensional electric field.

Part I: Exploring a Computer Simulated Electric Field

1. The field due to a line charge

a. What is a line of charge?

Introduction

Imagine rubbing a long thin plastic rod with fur so that it becomes charged *uniformly* along its entire length. Such a charge distribution can be characterized by a single number, its *linear charge density*, λ. It quantifies the amount of charge per unit length along the rod. We will simulate such a charge distribution in *EM Field* by placing point charges at equal distances along a line.

Procedure

- Open the program *EM Field* and maximize the window. From the menu bar select Sources/3D Point Charges. To help place charges uniformly in a line, from the menu bar select Display/Show Grid and then Display/Constrain to grid.

- As shown in the picture in the margin, create a uniform line of charge along the fourth row of grid points by repeatedly dragging +1 charges onto the grid. (Caution: The program will not allow you to place charges on the first and last grid points – so don't. If you ignore this caution and try to anyway, the charge will automatically snap to the second (or penultimate) grid point, possibly creating an unwanted pile-up on those points.)
- Consider how you could increase the average linear charge density of this line of point charges. There are two ways. One would be to increase the amount of charge on each grid point from 1 to 2 units. Describe a second way to increase the charge density without changing the magnitude of the point charges.

b. Electric field strength versus distance from a line charge

Introduction

You know that the magnitude of the electric field decreases with the square of the distance from a point charge; symbolically, $E \propto 1/r^2$. Does this pattern hold true as you move away from a line of charge? In this section you will perform a quantitative investigation of the field magnitude's dependence on distance from a uniform line of charge.

Procedure

- Drop field vectors 1, 2 and 4 grid units from the line of charge having a density of +1 units of charge per grid point. Measure the lengths of each field vector with a ruler and record your results in *Table 1*. Create a new line of charge having a density of +2 units of charge per grid point, and then repeat the measurements.

+1 charge units per grid pt.		+2 charge units per grid pt.	
Distance (grid units)	Field Vector Length (cm)	Distance (grid units)	Field Vector Length (cm)
1		1	
2		2	
4		4	

Table 1

- Use the program *Graphical Analysis* to determine the power law behavior of each fields' dependence on distance by fitting curves to E versus r using the functional form y = A*x^B. What is B to *two* significant digits for each? (You do not have to print the graphs.)

  Based on this result, if distance from the rod is doubled, by what factor will the field decrease?

  Comparing your results for the two lines of charge, one having double the charge density of the other, how does the power law behavior depend on linear charge density? Justify your answer.

- *EM Field* allows you to view a line charge from one end. Select *Sources>2D charged rod* from the menu bar. Place a rod near the bottom of the screen. Drop a few field vectors to find out how the electric field depends on the distance and direction from the rod. Are your results consistent with the results from above? Justify your answer.

Conclusions

Based on your observations and analysis does the field strength as you move away from a line charge decrease more rapidly, less rapidly or just as rapidly as when moving away from a point charge? Justify your answer.

2. Gauss' Law

Introduction

Gauss' Law is one of the most powerful, yet mystifying, statements in physics. Its mathematical form is,

$$\oint \vec{E} \bullet d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

In words, Gauss' law states that if you enclose a charge in a closed mathematical surface, then the electric flux integrated over that surface is proportional to the charge enclosed by that surface. The electric flux is defined as the product of the component of the electric field perpendicular to the surface times the area of the surface. What is the meaning of flux and Gauss' law? *EM Field* will help you get a feeling for the meaning of both. For a brief explanation of how *EM Field* shows the flux through a surface, select *Option>How we display Gauss' law* from the menu bar. Read the text, clicking on the screen for each new graph.

a. Flux from a single charged rod

Procedure

- Select a 2D charged rod with charge density +3 and place it in the middle of the screen. (You should still be in the "end-on" mode for viewing the charged rod.)
- Follow these steps to draw a Gaussian surface around the rod. First select *Field and Potential>Flux and Gauss' law* from the menu bar. Then use your mouse to draw a circle with a radius one or two grid units around the rod. When the cursor is near the starting point you can release the button and let the program complete the circle for you. Remember that this circle is actually the end view of a long cylinder enclosing the rod. Note that the electric field vector was always (approximately) perpendicular to the surface. The gray bars outside the circle represent the flux through each strip of the cylinder. "Q = 3" means that the value of the charge enclosed in the cylinder is three units.
- Draw a second circle with a much larger radius around the rod. The program again says "Q = 3," which means the total flux was the same as in the previous case, even though the electric field was much smaller at the surface.

  Explain how the total flux intercepted by each surface can be the same. (You must not answer, even though true, "Because the same amount of charge was enclosed." This is not an explanation. You should consider the result, Q = 3, as just that, a *result*, the endpoint of a calculation by the computer of the net flux intercepted by the Gaussian surface, which by Gauss' Law is proportional to the net charge enclosed. In expanded form, you are being asked, "Even though the electric field at each Gaussian surface is very different, one much smaller than the other, how is it that the integration to determine the net flux results in the same answer, indicating that the charge enclosed in each case, is indeed, the same?" To explain by saying, "Because the charge enclosed is the same", is to explain the end result with the result itself; and therefore, incorrect. Ask your instructor if you are confused. In many of the following questions you will have to answer in the same fashion, having to *avoid* referring to the amount of charge enclosed as an explanation.)

Explanation:

- Select *Sources>Add more charges* from the menu bar. Place a rod with charge density -3 in the center of the two circles and remove the +3 rod from the screen. What is different about the flux and how is it represented by the program?

- Remove the -3 rod from the screen and place a +3 rod in the center of the circles again. Select *Display>Unconstrain* so that you can place the rod anywhere on the screen. Now move it near the surface, but still inside the inner circle. The flux, and field, near the charge is much larger, but the total flux is still given by "$Q = 3$." Explain how this can be. (Again, you must not answer, "Because the charge enclosed is still the same.")

- Finally, move the +3 rod outside the inner circle. The program says that the enclosed charge, and thus the total flux through the inner surface, is now zero. Explain how that comes about. (Again, do not answer, "Because the charge enclosed is zero.")

Part II: Exploring a Real 2-dimensional Electric Field

Introduction

In this experiment you will create an electric field that is easy to map. To create the electric field you will clamp two small metal washers to opposite sides of a sheet of black "construction" paper. A potential difference of 12 V will be maintained between the washers by connecting them to the power supply as shown in *Figure 1*.

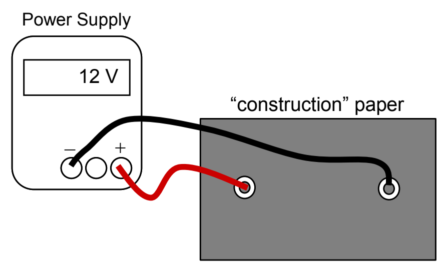


Figure 1: Establishing an electric field that can be mapped

The potential difference arises because the power supply *charges* the washers, one positive and one negative. This situation, reminiscent of an electric dipole, establishes an electric field.

The "construction" paper is not ordinary craft paper. It has been manufactured to be somewhat conductive. As a result, there is actually a small current (charge flow) in the paper. Current is not required to establish an electric field, but in this activity, having the conductivity of the paper "tuned" so that a small current is present has a number of desirable consequences.

i.) Because charge can flow we can be sure that the *direction* of the electric field at any point in the paper is in the *plane* of the paper. If it were not, that is, if there was a vertical component to the field, a vertical qE force would drive charges within the paper to the broad surfaces of the paper where they would pile-up, but only until the field they themselves produce exactly cancelled the vertical field. In other words, a steady state is reached in which the net field is entirely in the plane of the paper. That means you will be mapping a 2D field, not a 3D field.

ii.) Your measuring device for this experiment is a common laboratory multimeter configured to measure potential difference (voltage). If the paper was highly non-conductive, even though an electric field would be established, this meter would be incapable of measuring the potential differences along the paper.

iii.) Closely related to the previous point is that, with conductive paper, touching the probe to the paper does not significantly alter the field that has been established. In other words, use of the measuring device will not significantly alter the very quantity it is being used to measure.

As you are learning, there are two common ways to map the electric field in a region. The first is to create a map of *electric field lines*. You will take up that task in Part 2 of this section. The second is to create a map of equipotential *surfaces*; that is, surfaces of constant electric potential. In two dimensions, as is the case here, this becomes a map of equipotential *lines*. You will take up that task next in Part 1 of this section. The two maps are related, because they represent the same electric field. Familiarity with that relationship is an important learning goal of this activity and this part of the course.

1. Equipotential lines

Introduction

When you make the connections in *Figure 1* and turn on the power supply you won't see or sense anything happening. But in fact, an intricate but invisible pattern will have been established in the paper. Here we will augment our inadequate senses in order to perceive, to map, the pattern.

In electrostatics one way to map the field due to a collection of charges is to assign a single number, the electric potential, to each point in space. The SI unit for electric potential is the Volt. In this activity you will measure and record some of these numbers as a way of mapping the pattern hidden in the paper.

Procedure

- Place a sheet of the black conductive paper on the mapping board that has two bolts through it. Use a sharp point to create clean punctures in the paper, rather than simply forcing it onto the bolts. The washer should be resting on the top face of the paper, not the bottom face. Tighten the wing nuts to hold the washers firmly against the paper. The nuts/washers provide two point-like electrodes in the plane of the paper.
- Connect the power supply as shown in *Figure 1* above. Turn it on and set the voltage to read about 12 V.
- Now connect the digital multimeter (DMM) and potential probe as shown in *Figure 2*. (The potential probe is the one with only one point. The dual-probe consisting of two single-probes fastened together will be used in a later experiment.) Set the DMM to read dc volts.

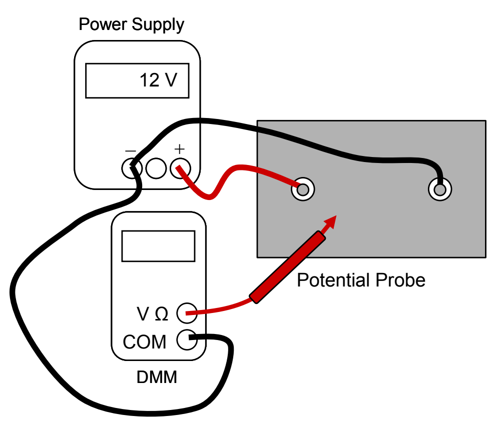


Figure 2: Wiring diagram for mapping equipotential lines

- Now touch the probe gently to the positive electrode. The multimeter and power supply readouts might not match. Adjust the power supply so that the *multimeter* reads 12.0 V since it is the device you will be using to make measurements.
- There is a value of electric potential associated with each point on the paper. Convince yourself of this by touching the probe to at least four random points. With a pencil mark each point and record the value of electric potential beside it.
- When dealing with electric potential only *differences* in potential have meaning. So the numbers you are recording only have meaning when taken with respect to

something else. The multimeter has been connected so that its COM jack is connected to the negative terminal of the power supply. As an example, this means if you measure a point as having an electric potential of 5.6 V, that point on the paper is 5.6 V higher in potential than the negative terminal of the power supply.

Touch the probe to the washer connected to the negative terminal of the supply. The meter should read 0.0 V. In other words, that washer is at the same potential as the negative terminal of the supply.

- The points you chose to probe were random, but the corresponding values of electric potential were not. There is a definite pattern to the numbers. Explore the pattern by moving the probe around the surface until you discover some order in the numbers.
- You may have realized that each point does *not* have a *unique* value of electric potential. Many points can have the same value. You'll also find that the many points corresponding to a particular value aren't randomly scattered, but rather, lie on a smooth continuous curve. This affords a useful way to illustrate the pattern in the numbers; that is, find enough points that correspond to a single value, until you can discern the shape of the equipotential line on which they lie.

  Give it a try. Find enough points that have a potential of 10 V that you can draw the continuous (probably closed) curve on which they lie.
- Repeat this procedure, finding the equipotential lines for 8 V, 6 V, 4 V and 2 V. Then you will have discovered the pattern hidden in the paper!
- To finish this section label all the equipotential lines with their potentials.

Questions

Finding values of electric potential is not just a way to map the hidden pattern. Potential differences have a physical meaning related to work and energy concepts. Mark two widely separated spots on the 4 V equipotential line and label them A and B. Mark one spot on the 8 V equipotential line and mark it C.

1. Electric fields exert forces on charges. It takes work to move charges against such a force. How much work, in Joules, would it take to move a +2 pC test charge from rest at point A to rest at point C? (Ask your friendly instructor for the simple formula if you haven't encountered it in lecture yet.)

2. How much work, in Joules, would it take to move a +2 pC test charge from rest at point A to rest at point B?

3. Based on your answers to these questions explain the physical significance of a line (or surface) of constant potential.

2. Field lines

Introduction

The next measurement is described in the following paragraphs. **Do not carry out the measurements until you have fully completed your prediction.**

An alternative to a map of equipotential lines is a map of electric field lines. You dealt with electric field line maps extensively in the previous lab. But they were presented to you. Here you will map them yourself.

Electric potential and electric field are related. As you are learning in class, and as you explored in the prelab assignment, it is a *derivative* relationship. The three components of the electric field at a point can be found by observing how rapidly the electric potential changes with location near the point in three orthogonal directions. Symbolically,

$$E_x = -\frac{\partial V}{\partial x} \qquad E_y = -\frac{\partial V}{\partial y} \qquad E_z = -\frac{\partial V}{\partial z}$$

There is nothing special about the x, y and z directions. The component of the electric field in any arbitrary direction, call it the s direction, is given by the same derivative relationship:

$$E_s = -\frac{\partial V}{\partial s}$$

This derivative can be "measured" approximately by making use of a ratio of *differences* to approximate the derivative. That is,

$$E_s = -\frac{\partial V}{\partial s} \approx -\frac{\Delta V}{\Delta s} \tag{1}$$

The numerator is a potential difference between two closely spaced points that can be measured with the multimeter. The denominator is simply the distance between the two points, and can be measured with a ruler – simple enough.

To conduct this measurement you will use the *field probe*. The field probe, which you should find on your table, consists of two sharp probes bound together and spaced about 1 cm apart. The field probe will be connected to the DMM, one wire to the black (COM) terminal and one wire to the red ("VΩ") terminal. Pressed against the black conductive paper, as shown in *Figure 3*, the field probe measures the potential *difference* between the two points with which the probe is in contact.

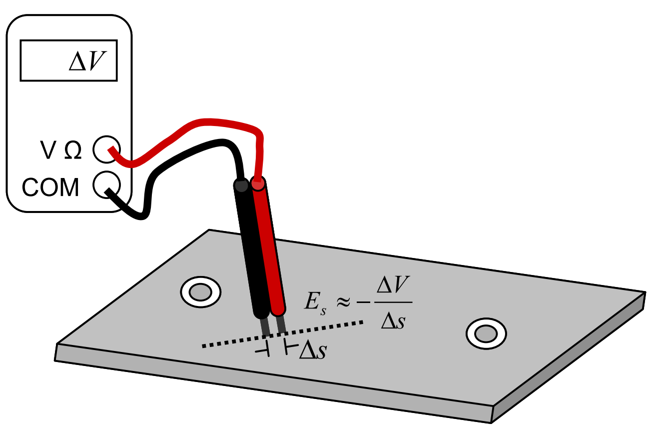


Figure 3: Measurement with the field probe

With the probe pressed against the paper one can imagine the line that passes through the points of both probes. The *component* of the electric field parallel to this line, at the point half-way-between the two probes, is well approximated *Equation 1*, where Δs is the 1 cm distance between the probes.

Prediction

Soon you will map out a *curved* field line. But first, from the symmetry of the system, you might guess correctly, that there is a *straight* field line that passes directly from the high potential washer to the low potential washer as shown in *Figure 4*. As you know, this means the electric field at any point on this line is parallel to this line and directed from high to low potential.

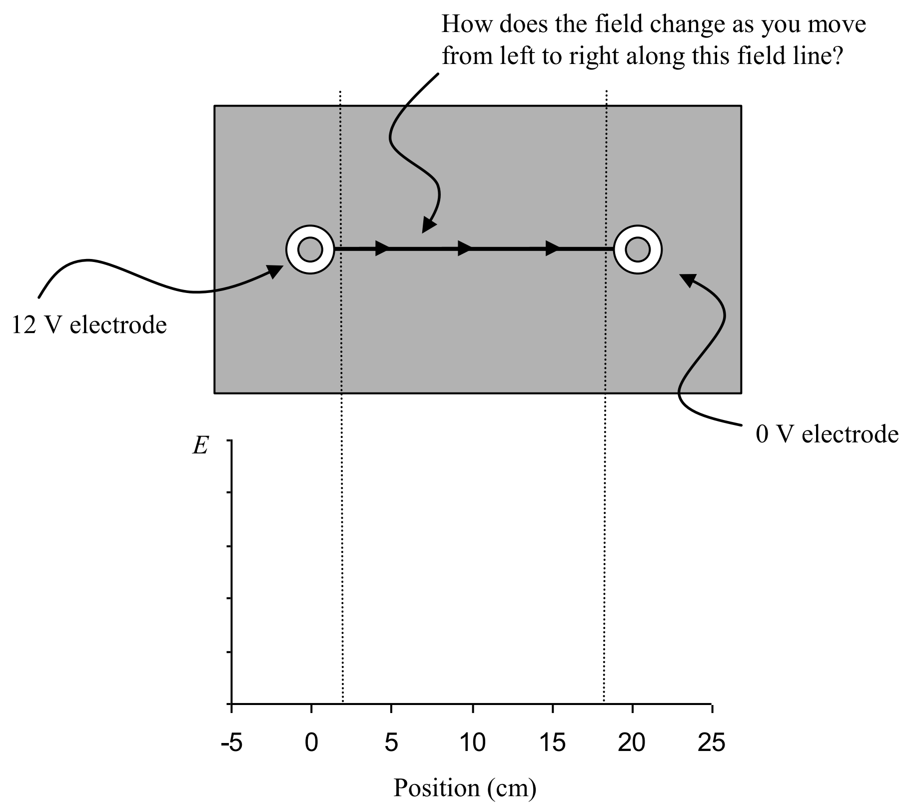


Figure 4: Examining the electric field along one field line

The distance between the centers of the washers along this line is about 20.3 cm and you will be measuring the electric field every 1 cm along that length starting just beyond the edge of the 12 V washer and ending just before the edge of the 0 V washer.

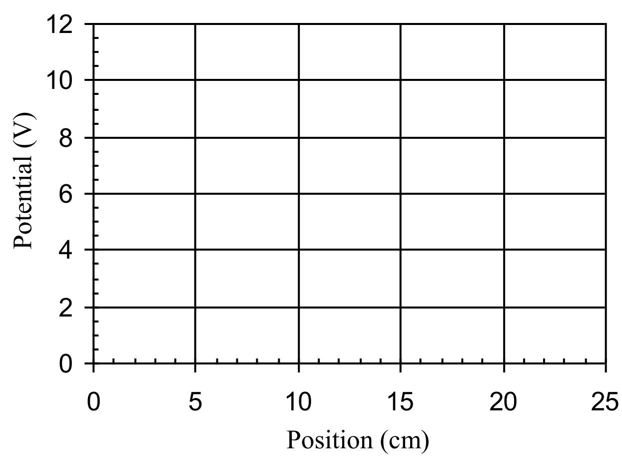
First, make a prediction. How do you think the electric field will change in magnitude along this line starting near the 12 V washer and ending near the 0 V washer? Make a qualitative sketch of your prediction between the dotted vertical lines on the graph in *Figure 4*. Remember, this is a graph of electric field, not electric potential.

Check your thinking

Before making the measurements check your thinking. To do so, first analyze some of the data you collected in the previous section when you mapped out equipotential lines. Along the straight line segment connecting the high potential washer to the low potential washer record the distances from the center of the 12 V washer to the points at which the potential is 10 V, 8 V, 6 V, 4 V and 2 V. (Two additional data points have already been filled in for you. The potential is 12 V from x = 0 cm out to the edge of the washer at about x = 0.5 cm, and it drops to the 0 V at the edge of the 0 V washer at about x = 19.8 cm.)

Position	**Potential**
0.5 cm	12 V
	10 V
	8 V
	6 V
	4 V
	2 V
19.8 cm	0 V

Now graph this data with position on the horizontal axis and potential on the vertical axis.

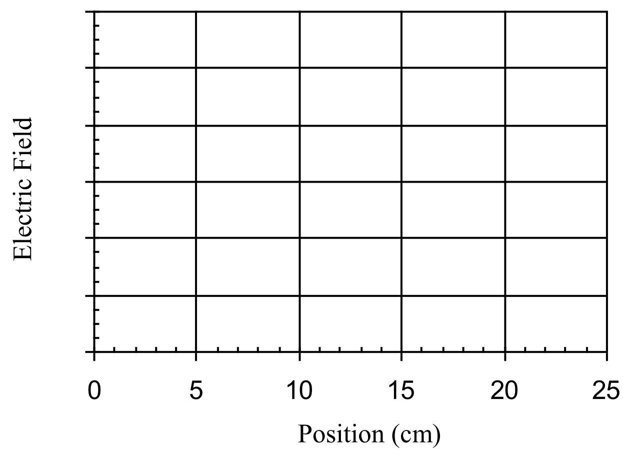


Draw a smooth curve that "best fits" the data. This does not mean draw straight-lines-to-connect-the-dots. It is a smooth curve. Nor does the smooth curve have to pass through every data point exactly. This is an attempt to interpret the pattern these few data points are conveying, an attempt to determine from the data the functional form of potential vs. position.

Due to the symmetry of the system the electric field is entirely in the x-direction at all points along the line connecting the electrodes. Therefore, the relationship between the electric field and the electric potential on this line is:

$$E_{along\,connecting\,line} = -\frac{\partial V}{\partial x}$$

This tells us, to determine how the electric field changes along the line that connects the washers, graph how rapidly the potential changes with respect to position along this line (with an overall negative sign). Do so! On the plot below sketch a graph of the negative of the derivative of the "best-fit" curve from the previous graph of potential vs. position.



You should consider this plot to be a "well-thought-out" prediction of what you are about to measure. Does it match the prediction you made earlier? Comment on any ways in which your earlier prediction was different.

Procedure

- Connect the field probe to the meter as described above and illustrated below in *Figure 5*. Have the DMM set to measure dc volts.

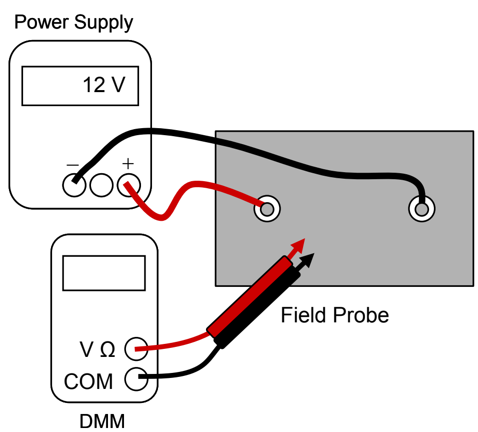


Figure 5: Wiring diagram for field line mapping

- Press the field probe against the paper so the black probe is just a few millimeters from the 12 V washer on the line that connects the two washers, with the red probe on the same line but, of course, 1 cm further along.
- If good contact has been made you should see a stable reading on the meter. In pencil, record this reading directly on the black paper near the probes. Don't forget to note the units. (The reading should be negative, as the meter gives $V_{\text{red}} - V_{\text{black}}$.)
- With small dots, mark the exact spots on the paper where the probes made contact.
- Move the probe so that the black probe is where the red probe was, and repeat this procedure. The red probe should again be on the straight field line and 1 cm further along.
- Repeat this procedure all the way out to the second electrode.
- Create a field line by connecting the dots. Place arrows on the field line in the direction of the field.

Analysis

- Fill the first column of the table below, by transferring your recorded data, with magnitude of potential difference $|\Delta V|$ using as many rows as you have measurements.

#	$\|\Delta V\|$ (V)	E (V/m)	x (m)
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			

- Fill column two by calculating the magnitude of the electric field that corresponds to each $|\Delta V|$ measurement.

- Fill column three with the position of the point to which the electric field in column two corresponds. Each position should be *measured separately with a ruler* from the center of the 12 V washer.
- Use *Graphical Analysis* to create a graph of E versus x . Format the graph with an appropriate title and axes labels. Print it and attach it to your lab report.

Conclusion

How does the actual electric field vs. position along the line connecting the washers compare with your initial prediction and with the second "well-thought-out" prediction?

Based on your graph, is the electric field constant between the two electrodes? If not, is there a region in which it is very nearly constant? Describe the region.

In what region, or regions, of the line between the two electrodes is the electric field strongest?

Is the electric field small near the 0 V electrode? Is there any conflict in your mind generated by the experimentally determined answer to this question? (rhetorical) If so, discuss the conflict with your instructor.

Is the electric field zero anywhere along the line between the two electrodes? If so, where?

Procedure (cont.)

- Many field lines should be generated to create a nice full map of the electric field. You will generate only one more. But unlike the last one, this one will be curved.

  Start by pressing the field probe against the paper with the black probe a few millimeters from the 12 V washer, but this time at an angle, positioned about 45° counterclockwise from the straight field line.

- The proper landing point for the red probe is not known (yet)! And it is grossly unacceptable to just guess, using intuition. This is a measurement!

  Your guiding principle follows this line of reasoning: the segment connecting the black to red probe is to be a segment of a new field line; the field is always directed along the tangent to the field line; therefore, this segment must be in the direction of the electric field. How do you determine the direction of the field?

  Pivot the red probe about the fixed black probe. Notice how the potential reading on the multimeter changes. Seek out a landing point for the red probe where the potential difference is its *maximum* (negative) value. *The direction of maximum drop in potential near a point is the direction of the electric field at that point.* Great!

- Make two small dots to mark the exact positions of the two probes. You do not have to record the potential drop itself.

- Now move the black probe to where the red probe was. Again, by pivoting the red probe, seek the landing point that gives the maximum drop in potential, and mark it. **Do not guess. Let the measurements guide you. The field pattern, invisible to your senses, is present in the paper. You are uncovering it, not guessing it.**

- Repeat this procedure until the measurements march you along to the 0 V electrode, or possibly, off the page. Draw a field line through the dots and attach some arrowheads. Now you have two!

Conclusion

Take a good look at the various points where an equipotential line and a field line meet on the construction paper. If you have good data, you should discern a general rule that is being followed. What is it?

Using the results from the second half of this lab and the answer to the previous question as a guide, draw a satisfyingly full set of field lines and equipotential lines in the space between the straight-segment and circle electrodes shown below. (Electrodes, made out of metal like the washers in your experiment, are surfaces of constant potential. Therefore, the 6 V line segment is 6V everywhere on the segment; the 0 V circle is 0V everywhere on the circle.)