

Name _____ Date _____ Time to Complete ____h ____m
 Partner _____ Course/Section ____/____ Grade _____

RC, RL and RLC circuits

Introduction

In this experiment we will investigate the behavior of circuits containing combinations of resistors, capacitors, and inductors. We will study the way voltages and currents change in these circuits when voltages are suddenly applied or removed. To change the voltage suddenly, a function generator will be used. In order to observe these rapid changes we will use an oscilloscope.

1. The square wave generator

Introduction

We can quickly charge and discharge a capacitor by using a function generator set to generate a square wave. The output of this voltage source is shown in *Figure 1*.

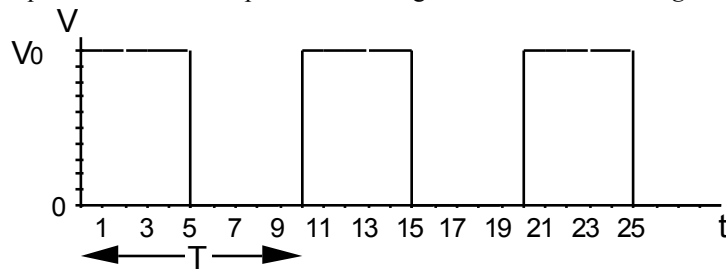


Figure 1: Output of square-wave generator

One control on the generator lets you vary the amplitude, V_0 . You can change the time period over which the cycle repeats itself, T , by adjusting the repetition frequency $f = 1/T$.

The generator is not an ideal voltage source because it has an internal resistance 50Ω . Thus, for purpose of analysis, the square-wave generator may be replaced by the two circuits shown in *Figure 2*. When the voltage is “on,” the circuit is a battery with an *EMF* of V_0 volts in series with a 50Ω resistor. When the voltage is “off,” the circuit is simply a 50Ω resistor.

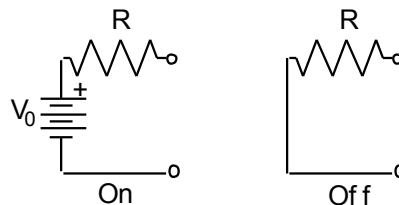


Figure 2: Square-wave generator equivalent circuit

Procedure

- To learn how to operate the oscilloscope and function generator, set the function generator for square wave output and connect the generator to the vertical input of the oscilloscope.
- Adjust the oscilloscope to obtain each of the patterns shown in *Figure 3*.
- Try changing the amplitude and repetition frequency of the generator and observe what corresponding changes are needed in the oscilloscope controls to keep the trace on the screen the same.
- Now set the function generator to a frequency of about 100 Hz. Observe the pattern and adjust the frequency until the period $T = 10.0$ ms.

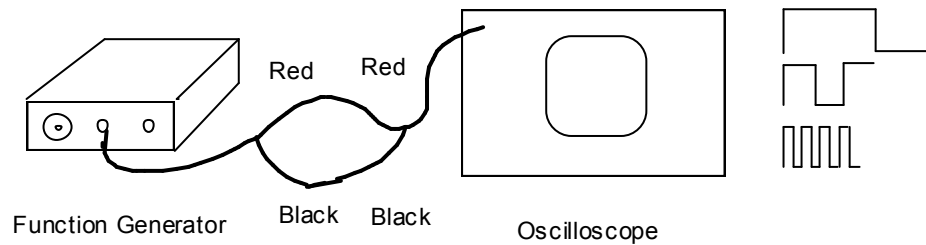


Figure 3: Observing the output of the square-wave generator

2. Resistance-capacitance circuits

Introduction

We have previously studied the behavior of capacitors and looked at the way a capacitor discharges through a resistor. Theory (see textbook) shows that for a capacitor, C , charging through a resistor, R , the voltage across the capacitor, V , varies with time according to

$$V(t) = V_0(1 - e^{-t/RC}) \quad (1)$$

where V_0 is the final steady-state voltage. When the same capacitor discharges through the same resistor,

$$V(t) = V_0 e^{-t/RC} \quad (2)$$

The product of the resistance and capacitance, RC , governs the time scale with which the changes take place. For this reason it is called the time constant, which we call τ (tau). It can be found indirectly by measuring the time required for the voltage to fall to $V_0/2$ (see *Figure 4* below). This time interval is called the half-life, $T_{1/2}$, and is given by the equation $T_{1/2} = (\ln 2)\tau$, so

$$\tau = \frac{T_{1/2}}{\ln 2} = \frac{T_{1/2}}{0.693} \quad (3)$$

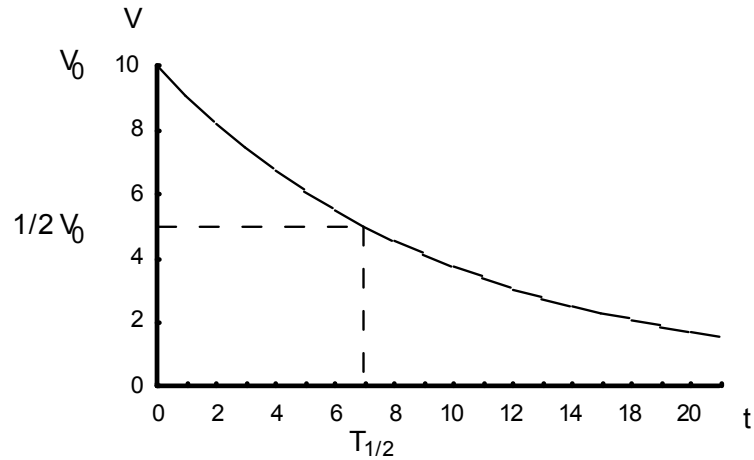


Figure 4: Discharge of a capacitor

Procedure

- Assemble the circuit shown in Figure 5.

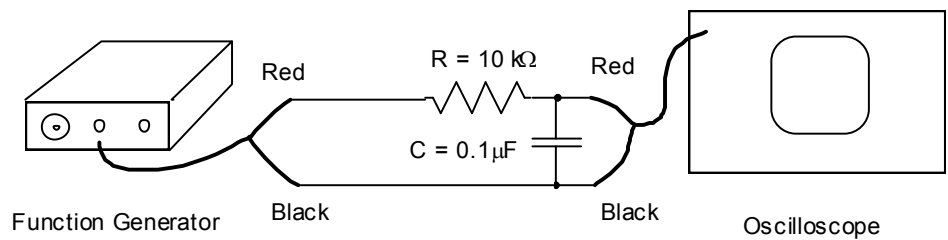


Figure 5: Investigating an RC circuit

- With initial values $R = 10 \text{ k}\Omega$, $C = 0.1 \text{ }\mu\text{F}$, and $f = 100 \text{ Hz}$, observe one period of the charge and discharge of the capacitor.
- Make sure the repetition frequency is low enough so that the voltage across the capacitor has time to reach its final values, V_0 and 0.
- Figure 6 shows one complete cycle of the input square-wave that is being applied across the resistor and capacitor. Superimpose on the square-wave a sketch of the waveform you observed, which illustrates the voltage across the capacitor as a function of time.

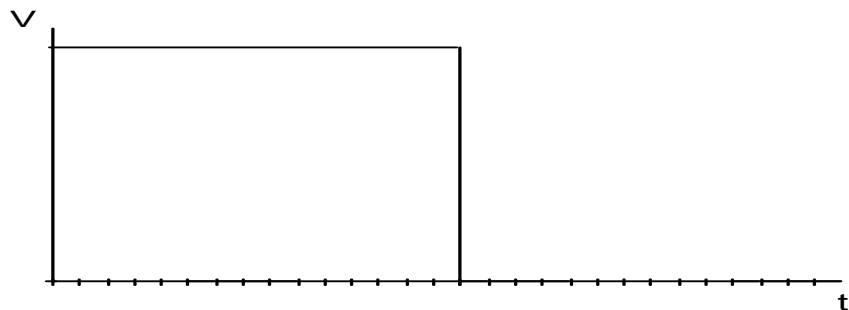


Figure 6: Capacitor voltage vs. time

- What is the largest voltage, V_0 , across the capacitor? What is the largest charge, q_0 , on the capacitor?

$V_0 =$ _____

$q_0 =$ _____

- Use the ohmmeter to measure R . (Recall that a resistor should be removed from the circuit before you measure its resistance with an ohmmeter.)

$R =$ _____

- To measure $T_{1/2}$ change oscilloscope gain (volts/cm) and sweep rate (ms/cm) until you have a large pattern on the screen, like the pattern shown in *Figure 7a*. Make sure the sweep speed is in the “calibrated” position so the time can be read off the x -axis.
- Center the pattern on the screen so that the horizontal axis is in the center of the pattern. That is, so that the waveform extends equal distances above and below the axis.
- Move the waveform to the right until the start of the discharge of the capacitor is on the vertical axis as shown in *Figure 7b*.
- The half-life is just the horizontal distance shown on *Figure 7b*.

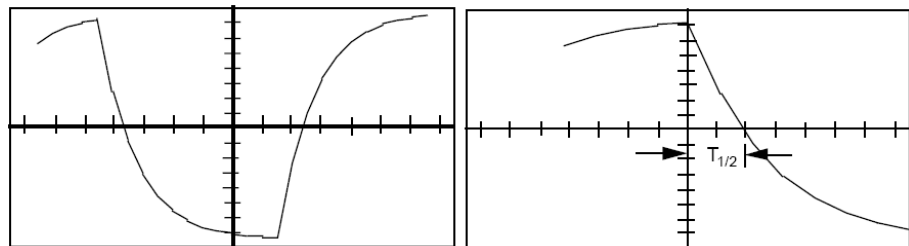


Figure 7a and b: Measuring the half-life

- Measure the half-life, $T_{1/2}$, and from this compute the time constant τ using *Equation 3*. Make sure to include units with your results

$T_{1/2} =$ _____

$\tau =$ _____

- You have just determined this circuit's time constant from the capacitor discharging curve. Theoretically, the time constant is given by the product of the resistance and capacitance in the circuit, RC . Compute RC from component values. Show your calculation in the space below. Note that, as described above, the square-wave generator has an internal resistance of 50Ω . Thus, the total resistance through which the RC circuit charges and discharges is $R + 50\Omega$.

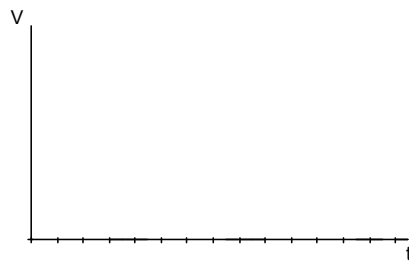
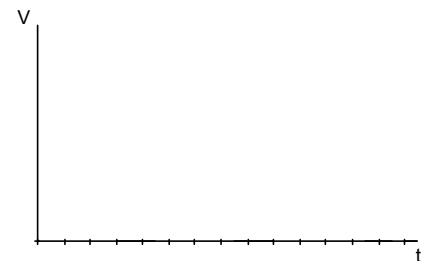
$\tau =$ _____

- When this calculation is carried out using ohms for resistance and Farads for capacitance, the product has units of seconds. Use dimensional analysis to show that this is indeed the case.

- Within the uncertainties of the tolerances (10%) of the resistor and capacitor, do your measurements support the equation $\tau = RC$? (If there is more than 20% disagreement, consult your instructor.)

- Although you have been told that the internal resistance of the function generator is 50Ω , let's say we had kept this piece of information from you. Without using an ohmmeter, outline a procedure for measuring the internal resistance of your function generator.

- Adjust the function generator to try different values of f and hence, T , while keeping τ fixed by not changing either R or C .
- On the left graph below sketch what you saw when the period T of the square wave was much less than the time constant, τ . On the right graph below sketch what you saw when the period T of the square wave was much greater than the time constant.

 $\tau \gg T$ $\tau =$ _____ $T =$ _____ $\tau \ll T$ $\tau =$ _____ $T =$ _____

3. Resistance-inductance circuits

Introduction

In this section we conduct a similar study of a circuit containing a resistor and an inductor, L . Consider the circuit shown in *Figure 8* below. The text shows that if we start with the battery connected to the LR circuit, after a long time the current reaches a steady-state value, $i_0 = V_0/R$.

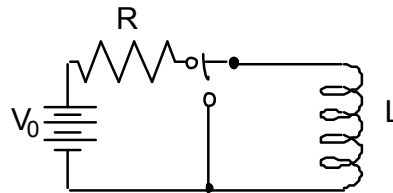


Figure 8: A model circuit with an inductor and resistor

If we call $t = 0$ the time when we suddenly throw the switch to remove the battery, allowing current to flow to ground, then current changes with time according to the equation

$$i(t) = i_0 e^{-(R/L)t} \quad (4)$$

If, at a new $t = 0$, we throw the switch so the battery is connected, the current increases according to the equation

$$i(t) = i_0 (1 - e^{-(R/L)t}) \quad (5)$$

The time constant for both equations is L/R and

$$\frac{L}{R} = \tau = \frac{T_{1/2}}{0.693} \quad (6)$$

We can find the current as a function of time by measuring the voltage across the resistor with the oscilloscope and using the relationship $i(t) = V(t)/R$. Note that what we would see first is the growth of current given by *Equation 5*, where the final current depends on the square-wave amplitude V_0 . Then, when the square wave drops to zero, the current decays according to *Equation 4*. The time constant should be the same in both cases.

Procedure

- Set up the circuit shown in *Figure 9* below.

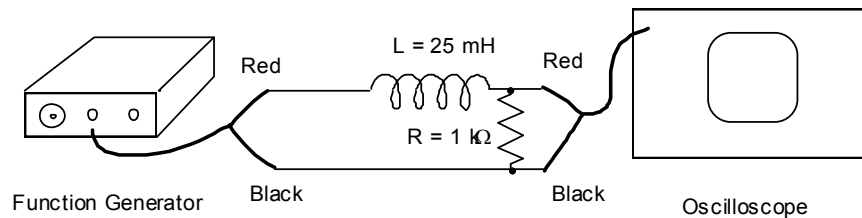
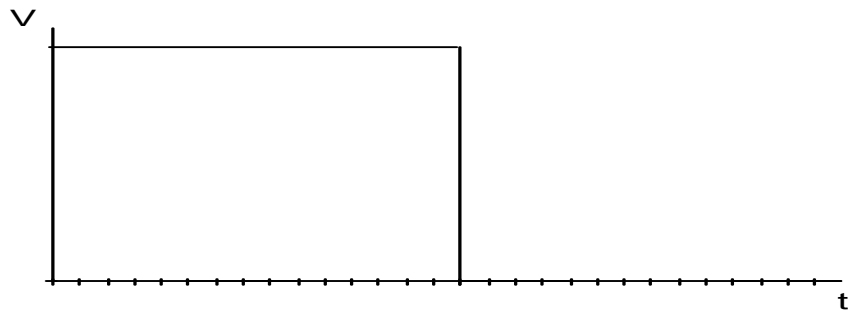


Figure 9: Investigating the LR circuit

- With initial values $R = 1\text{k}\Omega$ and $L = 25\text{mH}$, set the oscilloscope to view one period of exponential growth and decay. Again, make sure that f is low enough for the current to reach its final values, i_0 and 0. Start with $f = 5\text{ kHz}$. Superimpose a sketch of the waveform you observe on the single cycle of the input square-wave shown below.



- What is the largest current through the inductor?

$$i_0 = \underline{\hspace{2cm}}$$

- Measure the half-life. From this value, compute the time constant.

$$T_{1/2} = \underline{\hspace{2cm}}$$

$$\tau = \underline{\hspace{2cm}}$$

- Measure the value of R and the dc resistance of the inductor with an ohmmeter. Finally add the internal resistance of the square-wave generator to obtain the total resistance. Compute the value of L/R from the components' values.

R (of resistor) = _____

R (of inductor) = _____

R (of function generator) = _____

R (total) = _____

$\tau = L/R =$ _____

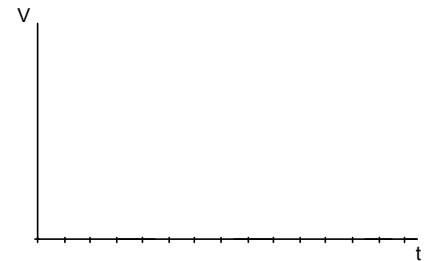
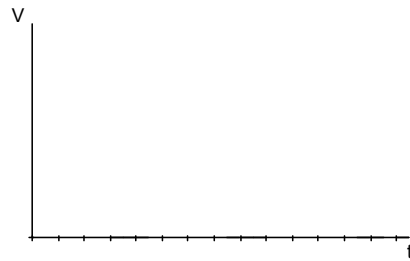
- Within the uncertainties of the manufacturing tolerances (10%) of the resistor and inductance, do your measurements support the equation $\tau = L/R$?

- When this calculation is carried out using ohms for resistance and Henries for inductance, the ratio has units of seconds. Use dimensional analysis to show that this is indeed the case.

- Adjust the function generator to try different values of f and hence, T , while keeping τ fixed by not changing either R or L .
- On the left graph below sketch what you saw when the period T of the square wave was much less than the time constant, τ . On the right graph below sketch what you saw when the period T of the square wave was much greater than the time constant.

$\tau \gg T$
 $\tau =$ _____
 $T =$ _____

$\tau \ll T$
 $\tau =$ _____
 $T =$ _____



4. Resistance-inductance-capacitance circuits

Introduction

As discussed in the textbook, a circuit containing an inductor and a capacitor, an *LC* circuit, is an electrical analog to a simple harmonic oscillator, consisting of a block on a spring fastened to a rigid wall.

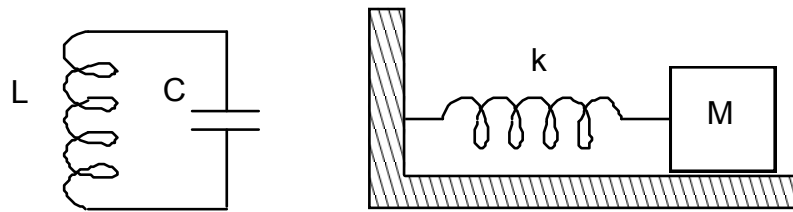


Figure 10: *LC* Circuit and its analog, a mechanical SHM System

In the same way that, in the mechanical system, energy can be in the form of kinetic energy of the block of mass M , or potential energy of the spring with spring constant k ; in the *LC* circuit energy can reside in the magnetic field of the inductor $U = \frac{1}{2} Li^2$, or the electric field of the capacitor, $U = \frac{1}{2} q^2 / C$. Both the current and the charge then change in a sinusoidal manner. The frequency of the oscillation is given by

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{7}$$

All circuits have some resistance, and in the same way frictional forces damp mechanical *SHM*, resistance causes energy loss (i^2R) which makes the charge decay in time.

$$q(t) = q_0 e^{-t/\tau} \cos(\omega_1 t) \tag{8}$$

$$\omega_1 = \left(\omega_0^2 - 1/\tau^2 \right)^{1/2} \tag{9}$$

where $\tau = 2L/R$ or

$$T_{1/2} = \ln 2(2L/R) = 0.693(2L/R) \quad (10)$$

For large τ the system is underdamped and the charge oscillates, taking a long time to return to zero.

Note from *Equation 9* that when $\omega_0^2 = 1/\tau^2$, ω_1 , which appears in the argument of the cosine function of *Equation 8*, is zero at all times. This condition is called critical damping. Critical damping occurs when $R = 2\sqrt{L/C}$.

When the resistor is larger than the critical value the system is overdamped. The charge actually takes longer to return to zero than in the critically damped case.

The decaying oscillations in the *LRC* circuit can be observed using the same technique as used to observe exponential decay. Again, a square-wave generator produces the same effect as a battery switched on and off periodically. The oscilloscope measures the voltage across *C* as a function of time.

a. Observing oscillations in a RLC circuit

Procedure

- Assemble the circuit of *Figure 11*. Use a small value of *R*, say, 47Ω . Be sure to reduce the signal generator frequency to 100 Hz or below so you can see the entire damped oscillation.

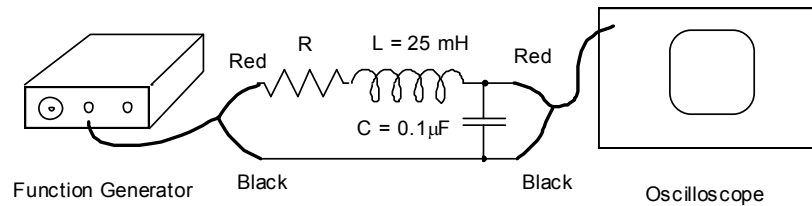


Figure 11: Investigating the LRC circuit

- Measure the period and calculate the frequency of the oscillations. (The period is NOT $0.01 \text{ s} = 1/100 \text{ Hz}$, the repetition frequency of the square wave.)

Measured period = _____

Calculated $f_1 =$ _____

$\omega_1 = 2\pi f_1 =$ _____

- Calculate ω_0 from component values.

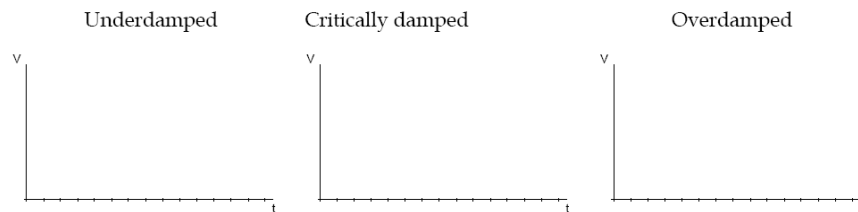
$\omega_0 = 1/\sqrt{LC} =$ _____

- Compare the ω_1 you measured with ω_0 that you calculated from component values. In theory, ω_1 , the damped frequency, is only slightly less than ω_0 , the undamped frequency, making this a valid comparison of theory with experiment.

b. Critical damping and overdamping in a RLC circuit

Procedure

- Note that in the equations for this circuit, R represents the sum of the resistance of the inductor, the internal resistance of the square-wave generator, 50Ω , and the resistance of the resistor.
- To study critical damping and overdamping, remove your fixed resistor and put in its place a $5\text{-k}\Omega$ variable resistor.
- Start with the variable resistor set to a small value of R . For small R you should see the oscillations that are characteristic of underdamping. On the first graph below sketch the waveform that appears on the oscilloscope.



- Increase R until critical damping is reached; that is, until the oscillations disappear. Sketch this curve on the middle graph above. Use the ohmmeter to measure the value of the variable resistor at critical damping. (Don't forget to disconnect the variable resistor from the circuit when measuring its resistance.)

R (variable resistor at critical damping) = _____

- Calculate the total resistance in the circuit at critical damping by adding the dc resistance of the inductor and 50Ω for the function generator to the result above.

R (total at critical damping) = _____

- Compare this value of the circuit resistance at critical damping to the predicted value for critical damping, $R = 2\sqrt{L/C}$.

- What happens to the waveform when the resistance is larger than the critical-damping value? Sketch your results on the rightmost graph above.

c. Underdamping in a RLC circuit

Introduction

When the circuit is underdamped, *Equation (8)* applies. This means that the amplitude of the oscillation will decay exponentially, with the time constant for the decay being:

$$\tau = \frac{2L}{R} \quad (11)$$

Recall that when an exponential decay is plotted on a semi-log scale the resulting graph is a straight line with a slope equal to $-1/\tau$.

You can find the slope of a line on a semi-log graph by identifying the two end points of the line. Note the time and voltage at each point t_1 and V_1 , t_2 and V_2 .

Calculate the natural log of the two voltages. Then,

$$\text{slope} = \frac{\ln(V_2) - \ln(V_1)}{t_2 - t_1} = -\frac{1}{\tau} \quad (12)$$

The following steps describe how to measure the time constant of the decay of the oscillations.

Procedure

- Adjust the variable resistor so that the circuit is underdamped and oscillates about seven or eight times before the oscillations become too small to be easily seen on the oscilloscope.
- Center the oscillation pattern vertically on the screen so that when the oscillations have decayed the line on the oscilloscope coincides with the time axis.

In the table below record the voltage of each oscillation peak, and the corresponding time for each peak. When your table is complete you should have six or seven sets of data recorded.

Peak Height (V)	Time ()

Table 1

- Create a semi-log graph of your data. (You can use Excel to create a semi-log graph.)
- Following the procedure described above, determine the time constant for your circuit. This is your experimental value for the time constant. Show your calculation and result here.

$$\tau_{\text{experiment}} = \underline{\hspace{2cm}}$$

- Remove the variable resistor from the circuit and measure its resistance. Add this value to the resistance of the square-wave generator (50 ohms) and the resistance of the inductor to get the total resistance of your circuit. Show your calculation and result here.

$$R_{\text{total}} = \underline{\hspace{2cm}}$$

- Using *Equation* (11) calculate the theoretical decay time constant for your circuit. Show your work.

$$\tau_{\text{theory}} = \underline{\hspace{2cm}}$$

- Compare this theoretical value to the experimental value you found above. They should agree within ten or twenty percent. If they do not, consult your instructor.