Lab 11: Geometric Optics

Part 1: Reflection and Refraction

With this experiment we begin the study of light and its interaction with various optical components. In this experiment we shall study the limit where the wave nature of light can be neglected and we can consider a train of light waves to be a straight line, or ray. Wave effects are usually unimportant when the size of the optical component is much larger than the wavelength of the light which is interacting with it. Situations where the wave nature of light must be taken into account will be studied in a later experiment.

In order to define terms, Figure 1 shows a light ray (in an optical medium having index of refraction \(n_1\)) incident from the left onto an interface separating the first optical medium from a second optical medium having index of refraction \(n_2\). The index of refraction of an optical medium is defined as the ratio of the speed of light in a vacuum (symbolized by “c”) to the speed of light in the optical medium:

\[
    n = \frac{c}{v} \quad \text{Equation 1.}
\]

In general, the speed of light in an optical medium depends on the wavelength of the light. Therefore the index of refraction will also depend on the wavelength. This dependence of index of refraction on wavelength, called dispersion, explains how a prism is able to spread a beam of white light into its component colors.

![Figure 1: Reflection and Refraction at a Surface](image)

As the figure shows, an incident ray of light will, in general, split into two parts as it hits the surface separating the two media. One part, the reflected ray, will remain in the first medium, and will make an angle (measured relative to the normal to the surface) that equals the angle the incident ray made with the normal. Expressed mathematically, the law of reflection says:

\[
    \theta_i = \theta_i' \quad \text{Equation 2.}
\]
The second part, the refracted, or transmitted, ray, proceeds on into the second medium. This ray makes an angle, \( \theta_2 \), measured relative to the normal line in the second medium, which is determined by the Law of Refraction (also known as Snell’s Law). Snell’s Law states that:

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad \text{Equation 3.} \]

Note: Figure 1 (previous page) represents a situation where \( n_2 > n_1 \), resulting in \( \theta_2 \) being smaller than \( \theta_1 \). If \( n_2 < n_1 \) the situation is reversed and \( \theta_2 \) becomes greater than \( \theta_1 \). This would be the case, for example, if a ray of light already in an optical medium such as glass or transparent plastic was exiting back into an optical medium such as air. Figure 2 depicts this situation:

Note that this means that if \( \theta_1 \) is allowed to get larger, \( \theta_2 \) will eventually approach 90°. But when \( \theta_2 = 90° \) it means that no refracted ray goes into the second medium. Rather the incident ray is completely reflected back into the first medium. This phenomenon is called total internal reflection. Clearly this can happen only when a ray is incident from a medium of higher index of refraction into a medium of lower index of refraction. Referring to Equation 3, we see that total internal reflection first occurs when \( \sin \theta_2 = 1.0 \). When this happens, \( \theta_1 \) is said to be at its critical value, \( \theta_{\text{crit}} \). This critical value of \( \theta_1 \) can therefore be found by setting \( \sin \theta_2 = 1 \) in Equation 3 and solving for \( \sin \theta_1 \). Remembering that \( \theta_1 \) is now \( \theta_{\text{crit}} \), the resulting equation is:

\[ \sin \theta_{\text{crit}} = \frac{n_2}{n_1} \quad \text{Equation 4.} \]

**Procedure:**

**Part 1a:**

Attach the optical table to the optics bench and place the semicircular plastic lens on the optical table. The center of the flat face of the lens should be aligned with the line on the table labeled “component”, and the normal line should intersect the center of the flat face. See Figure 3 on the next page.
Attach the light source to the optics bench and orient the light source so that a single ray of white light leaves the source and shines along the normal line on the optical table. The system should be aligned so that this incident ray (A) strikes the center of the flat face of the semicircular lens. The angle of incidence ($\theta_1$) can then be varied by rotating the optical table. Note that the refracted ray (C) should not change direction as it exits the circular face of the lens. Be sure that you understand why this is the case.

Now vary the angle of incidence from $0^\circ$ to $90^\circ$ in $10^\circ$ increments. For each angle of incidence, measure and record the angles of incidence, reflection, and refraction and make a table of your data.

Do you find that, within the uncertainty in your measurements (you should estimate this--are your measurements accurate within $\pm 1^\circ$, $2^\circ$, or what?), the angle of reflection equals the angle of incidence as predicted by the law of reflection? If not, redo the experiment taking greater care in aligning the apparatus and measuring the angles.

Now use Snell’s Law (Equation 3) to determine the index of refraction of the plastic from which your lens is made. Proceed as follows:

1. For each angle of incidence, calculate $\sin \theta_1$ and $\sin \theta_2$.

2. For each angle of incidence, use the results of step 1, and Equation 3, to calculate $n_2$. Assume that $n_1$ (the index of refraction of air) is 1.00.

3. Calculate and report the average and standard deviation of your results. The average of all your calculated values represents the most reliable value for your measurement of index of refraction, and the standard deviation is a measure of the uncertainty in your average value.

4. According to the manufacturer, the index of refraction of the plastic from which the hemispherical lens is made is 1.50. Does your average value agree with this value to within the standard deviation of your measurements? If not, redo this portion of the experiment, taking greater care with your measurements.
Part 1b:

Now arrange the apparatus as shown in Figure 4:

![Figure 4](image)

Be certain that you arrange your apparatus so that the incident ray (A) undergoes no deviation at the cylindrical surface of the lens, and so that the incident ray inside the lens hits the flat surface at its center. You are now seeing refraction at the flat surface as the ray passes into a medium (air) having a lower index of refraction than the plastic, so the refracted ray (C) is bent away from the normal as it passes from the plastic into the air.

Repeat the procedure of Part 1a, and make a second determination of the index of refraction of the plastic. (Note that at some value of the angle of incidence, the refracted ray will disappear. This value of the angle of incidence is the critical angle. Make several careful measurements of this angle for use later.) Does your result from Part 1b agree with your result from Part 1a within their combined standard deviations? If not, repeat Part 1b, taking greater care with your measurements.

Now calculate the average and standard deviation of your measurements of the critical angle. Use Equation 4 to calculate the index of refraction of the plastic based on your measurement of this angle. Does this result agree with the results of Parts 1a and 1b within their standard deviations? If not, repeat your measurements more carefully.

Part 2: Image Formation by Lenses

From the point of view of optics, an object is a source of light rays. These rays may exist because the object itself emits them (such as, for example, light bulbs, stars, or the sun) or because the object reflects light from a source other than itself. Most objects (for example people, trees, rocks, etc) fall into this second category. In either case, under certain conditions, a lens can form an image from the light rays coming from the object. The lens bends, or refracts, the light rays, causing them to interact in such a way as to form the image of the object from which they came. Careful measurements of the distance between the object and the lens (the object distance, \( p \)) and the distance between the lens and the image (the image distance, \( I \))
can be used to determine the focal length, \( f \) of the lens. The focal length is a measure of the imaging power of the lens. In this part of the experiment we will study how a lens produces an image.

**Ray Tracing:**

A simple way to understand how a lens works is to follow a procedure called *ray tracing*. A ray is an imaginary straight line that traces the path of the light. By following the paths of two or three specially selected rays coming from the same point on the object, it is possible to construct a diagram which visually depicts the creation of the *image point* of that point on the object. As they pass through the lens, the rays will be refracted at both the front and rear surfaces of the lens. After passing through the lens, the rays will, in general, intersect at some point in space. This point is the image point we are looking for. If the lens is not too thick, it is possible to simplify the ray tracing process by assuming that the ray changes direction not at the surfaces of the lens, but rather at a plane through the middle of the lens. If the lens is sufficiently thin (such at the lenses we will be using in the lab), this approximation does not introduce and errors.

Your textbook gives a discussion of ray tracing, along with several examples. Be sure to review this information before coming to lab. Be sure that you clearly understand how to construct accurate ray diagrams for both convex (positive) and concave (negative) lenses for all of the categories of images these lenses are capable of producing.

**2a: Focal Length of a convex lens**

For thin lenses, object distance, \( p \), image distance, \( i \), and focal length, \( f \), are related by a simple equation:

\[
\frac{1}{p} + \frac{1}{i} = \frac{1}{f}
\]

Equation 5

The ratio of the height of the image, \( h_I \), to the height of the object, \( h_O \), is called the *transverse magnification*, \( m \). It can be calculated from:

\[
\frac{h_I}{h_O} = m = -\frac{i}{p}
\]

Equation 6

You will now make measurements involving the focal length of a positive lens by three different methods, each method intended not just to gather data and do a calculation, but also to familiarize you with the characteristics of the image produced in each case.

**Method A:**

If the object is far from the lens, the value of \( 1/p \) becomes so small that \( 1/i \) approximately equals \( 1/f \). Or, in other words, \( i \) approximately equals \( f \). The image produced will, in general, be quite small and will be located approximately at the focal point of the lens.
Procedure:

1. Remove the light source table from the optics bench and mount the lens and viewscreen on the optical bench.

2. Orient the bench so that the image of a distant object (a lightbulb at the far end of the lab, for example) appears on the screen.

3. Adjust the location of the screen until the image is sharp and clear. Measure and record the distance between the lens and the screen. This image distance approximately equals the lens’s focal length.

4. Draw a ray diagram depicting how this image is formed on the screen.

Method B:

When the object distance is twice the focal length of the lens, Equation 5 can be solved to show that the image distance is also twice the focal length. The transverse magnification is then -1. This means that the image is the same size as the object, but is inverted.

Procedure:

1. Solve Equation 5 and show that if \( p = 2f \) then \( i = 2f \) also.

2. Now mount the light source near one end of the optical bench and orient the source so that the face with the crossed arrows on it points toward the lens. Using your approximate focal length from Method A as a guide, locate the screen approximately 4\( f \) from the light source, and place the lens halfway between them.

3. Carefully adjust the positions of the lens and the screen until you have a clear image that is exactly the same size as the object (use the vernier caliper at your lab station to measure the size of the image and the size of the object), and so that the object distance equals the image distance.

4. Measure and record your values for object distance and image distance and use them to calculate the focal length of your lens.

5. Draw an accurate ray diagram depicting how the image is formed in this case.
Method C:

For an arbitrarily chosen object distance, and a lens of known focal length, Equation 5 can be used to predict the location of the image. Your results from Method B should give you an accurate and reliable value for the focal length of your lens. There are three possible ranges in which the object distance can fall: \( p > 2f, f < p < 2f, \) and \( p < f. \) For each of these ranges, do the following:

Procedure:

1. Choose an object distance and use Equation 5 to calculate a predicted value for the corresponding image distance.

2. Setup your optics bench using your chosen value of object distance and your calculated value of image distance and see whether you do indeed get an image at the predicted location. If so, measure and record your experimental values of \( p \) and \( i \) and compare the predicted and experimental values of the image distance.

3. Draw an accurate ray diagram depicting the formation of the image in each case.
# Geometric Optics

## Worksheet

### Part 1: Reflection and Refraction

#### Part 1a:

<table>
<thead>
<tr>
<th>Incident Angle ($\theta_1$)</th>
<th>Reflected Angle ($\theta'_1$)</th>
<th>Refracted Angle ($\theta_2$)</th>
<th>$\sin(\theta_1)$</th>
<th>$\sin(\theta_2)$</th>
<th>Index of refraction (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Index of refraction: Average(n)______________________ +/-(Max(n)-Min(n))______________

#### Part 1b:

<table>
<thead>
<tr>
<th>Incident Angle ($\theta_1$)</th>
<th>Reflected Angle ($\theta'_1$)</th>
<th>Refracted Angle ($\theta_2$)</th>
<th>$\sin(\theta_1)$</th>
<th>$\sin(\theta_2)$</th>
<th>Index of refraction (n)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Index of refraction: Average(n)______________________ +/-(Max(n)-Min(n))______________

Critical angle: _____________

Index of refraction from critical angle measurements: __________________.
Conclusions:

1. Explain why the refracted ray is not deviated at the cylindrical surface of the lens in part 1a.

2. Based on your understanding of Snell’s Law, show mathematically why angle Z equals angle X in the diagram below.
Part 2: Image Formation by Lenses

Method A:
Approximate focal length__________________
Ray Diagram:

Method B:
Solve Equation 5:

Object Distance__________________ Image Distance__________________
Calculated focal length:________________
Ray Diagram:
Method C:

1. $p > 2f$:

Object distance________________________
Predicted image distance________________
Measured image distance________________
Ray diagram:

2. $f < p < 2f$:

Object distance________________________
Predicted image distance________________
Measured image distance________________
Ray diagram:

3. $p < f$:

Object distance________________________
Predicted image distance________________
Measured image distance________________
Ray diagram:
Conclusion:

Summarize your understanding of image formation by lenses by completing the following table. Under what circumstances is the image larger than the object? Smaller? Same size? Are there circumstances when no (real) image forms at all? What about relative image size and orientation?

<table>
<thead>
<tr>
<th>Object Distance</th>
<th>Image Size? (magnified, diminished)</th>
<th>Image Orientation? (upright, inverted)</th>
<th>Image? (real, inverted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>p &gt; 2f</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f &lt; p &lt; 2f</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p &lt; f</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>p = infinity</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>