Interference and Diffraction of Light

Introduction
If you are wondering what experimental evidence exists for the claim “light is a wave” look no further. In this lab you will directly observe interference patterns and diffraction patterns of light. These patterns are hallmarks of wave phenomena. The patterns are familiar from other systems that exhibit wave behavior, such as water waves.

Reflection by mirrors and refraction by prisms and lenses, as you studied in the previous lab, can be analyzed using a simple ray model of light, in which the wave nature of light is ignored. In contrast, interference and diffraction patterns, which appear when light encounters small apertures or obstructions, cannot be explained with the ray model; the wave nature of light must be explicitly taken into account in order to understand them. This lab provides an opportunity to directly observe these hallmark wave phenomena and to further your conceptual understanding of interference and diffraction.

In addition, beyond this basic physics, you will also learn about an important application of light interference in technology – its use in precision measurements. In fact, some exquisitely sensitive measurements employ light interference. If you are interested, research the Michelson-Morley experiment as a famous “old” example in the history of physics, or LIGO as an extraordinary example of an on-going experiment, one that is used to detect gravitational waves.

You will conduct your own precision measurement, one to determine the diameter of a thin wire or strand of hair, using light interference. Parts 1 and 2, in which you observe light having passed through two narrow closely-spaced slits and through a single narrow slit, respectively, will prepare you for this precision measurement in Part 3.

Questions
According to the introduction, under what circumstances are interference and diffraction patterns observed?

According to the introduction, of what use are light interference patterns in technology?

According to the introduction, what will you attempt to measure in the last part of the lab?
1. Double slit interference

Introduction

A single light source, the red light of a laser, will be used throughout this lab. We will consider this light to be monochromatic; that is, consisting of a single wavelength. To conduct the precision measurement in Part 3 the wavelength of your laser light must be known. At the start of this first activity the wavelength should be considered unknown, but a procedure will be described and implemented to measure it.

Activity #1 Goal: Determine the wavelength of the laser’s red beam.

In this first activity the red laser light will be aimed at a pair of narrow closely spaced slits, and the light emanating from them will be projected on a distant view screen. The pattern of light on the view screen (called an interference pattern, or fringe pattern) will be observed and analyzed. This is often referred to as a Young’s Double Slit apparatus. Features of the pattern depend on the light wavelength and therefore a careful analysis of the pattern will make possible a calculation of it.

A top down diagram of the apparatus geometry is shown below in Figure 1. The slit separation \( d \) is much less than the view screen distance \( D \). A point on the view screen, such a \( P \), can be identified either by the angle \( \theta \) made with respect to the central axis or by the length \( y \), measured from the center of the screen at \( P_0 \), as shown in the diagram. (It is easy enough to convert from one to the other because \( D \tan \theta = y \).)

When the laser is on, the two slits can be modeled as two coherent sources of light. The light waves emanating from these two sources combine to form an interference pattern. At the view screen, total constructive interference occurs at points where the two combining waves are in phase. If the waves are in phase at the slits, one point of constructive interference occurs where the path lengths from slits to screen are identical; in this geometry that point is on the central axis at the spot marked \( P_0 \). But total constructive interference also occurs at other points, those where the path lengths differ by a distance equal to an integral number (e.g. \( 1, 2, 3, \ldots \)) of wavelengths. Using these conditions and the geometry of the system it can be shown that the angles at which total constructive interference occur satisfy the equation:

\[
d \sin \theta_m = m \lambda \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots
\]

In this equation \( \lambda \) is the light’s wavelength and \( m \) is an integer index that can take on the values \( 0, \pm 1, \pm 2, \pm 3, \ldots \). The positive and negative values refer to maxima, symmetrically positioned, on one side or the other of the central axis. In words, for example, one would...
say that $\theta_i$ identifies the angle at which the third order ($m = 3$) maximum of the interference pattern occurs.

Later in this section you will tabulate the positions of these maxima, by measuring their locations, but you won’t be measuring the angles; instead, you will measure the distance of each maximum from the center of the pattern along the view screen, with a ruler or Vernier calipers. These measurements are like the length $y$ from Fig. 1. To relate Eq. 1 directly to these $y_m$ measurements we will invoke the “small angle approximation:” if the $\theta_m$ are small, then $\tan \theta_m \approx \sin \theta_m$. It has already been pointed out, based on the geometry shown in Fig. 1, that $\tan \theta = y/D$. Thus, in the small angle approximation, $\sin \theta \approx y/D$. By substituting this in Eq. 1 we arrive at the important relationship,

$$y_m = \frac{m\lambda D}{d} \quad m = 0, \pm 1, \pm 2, \pm 3, \ldots$$

(2)

**Procedure**

- Secure the view screen on the track so that its front face is exactly at the 0 mm mark of the tape measure.

  **Caution!!! Never look directly into a laser beam!!!**

- Position the laser at the very end of the optics track opposite the view screen so that it points toward the view screen. (You may plug it in and turn it on to make sure it works. But then turn it off again.)

- Turn the wheel on the Multiple Slit Set so as to center the double slits having a nominal slit width $a = 0.04$ mm and a slit separation $d = 0.25$ mm.

- Secure the Multiple Slit apparatus on the optics track far from the view screen and close to the laser. The slits-to-view screen distance $D$ is an important parameter. You should aim to have $D$ at least 1000 mm. As an aid to precise positioning note that the horizontal distance from the slits to the near edge of the positioning pointer at the base of the Multiple Slit apparatus is 21 mm as shown in Figure 2 below.

Use this fact and the tape measure to precisely determine $D$ and record the value below in millimeters. Also estimate an uncertainty in this value.
Turn on the laser and make a few final adjustments to obtain a nice bright and horizontal interference pattern. On the back of the laser you will find screws to adjust the vertical and horizontal position of the laser beam. Adjust them so that the beam is centered on the slits producing the brightest pattern possible.

The pattern you observe on the view screen should be a horizontal line of alternating bright red and dark spots, or maxima and minima, something like that shown in Figure 3 below.

You will be marking locations of maxima (bright red spots) in the interference pattern. Instead of marking the view screen, attach a piece of masking tape across the width of the view screen so the interference pattern falls on the masking tape.

Identify the bright red spot at the very center of the pattern and make a pencil mark near it (slightly above or below it) on the tape. This is the central $m = 0$ maximum for which the angle and position are zero ($\theta_0 = 0$, $y_0 = 0$).

Continue in this fashion by marking the location of the next 16 maxima, $m = 1$ to $m = 16$. The maxima should be about evenly spaced. If there is a gap, suggesting a “missing” maximum, mark it as well. (You only have to mark maxima to one side of center. The pattern is symmetric.)

We are interested in both the position and relative brightness of the maxima. Note how the brightness of the maxima changes as you scan away from the center of the pattern. Although we don’t have a brightness (intensity) meter to measure quantitatively the intensity of each maximum we can make a crude attempt to quantify the trend by using our eye/brain system as an intensity meter. (This is problematic for any quantitative analysis because our eye/brain system is not a linear intensity sensor. In other words, your brain may perceive one maximum to be about twice as bright as another, but in reality the relative intensity is much different.)

Nonetheless, let’s say the intensity of the central maximum, which is the brightest, is 100 in some arbitrary units. In the table on the next page, record the relative brightness of the 16 maxima that were marked by assigning each a numerical value of intensity relative to the central maximum. For example, if you think some maximum is half as bright as the central maximum record an intensity of 50. If a “missing” maximum was marked, assign it an intensity value of zero.
• When finished, remove the tape from the view screen, and without wrinkling it, tape it in the box below.

![Tape tape here.]

• Use a ruler or the Vernier calipers, as shown in Figure 4, to measure and record the locations of all the marked maxima with respect to the central maximum. Record your measurements in millimeters in the “$y_m$” column of the table below.

![Figure 4: Calipers shown measuring position of the 4th order maximum]

<table>
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<tr>
<th>$m$</th>
<th>Intensity (Arbitrary Units)</th>
<th>$y_m$ (mm)</th>
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• Use your position and intensity data to sketch a graph of intensity versus position as you move away from the central maximum at y = 0.

Although you should certainly plot points, your final graph should be a smooth curve. When you draw in the smooth curve don’t forget to incorporate the fact that the intensity falls to zero, or nearly so, in-between maxima.

Great! Certainly any theory of interference and diffraction should be able to reproduce this function theoretically. Is there a function that has this shape? And how would it be derived from fundamental principles? (Rhetorical)

• Mark and label at least three points on the graph where the interference is CONSTRUCTIVE. The interfering waves are in phase, or nearly so, at these points.

• Mark and label at least three points on the graph where the interference is DESTRUCTIVE. The interfering waves are out of phase, or nearly so, at these points.

• Hopefully you haven’t forgotten the goal of Part 1. We want to determine the wavelength of the laser light! We’ll do that now. You should be able to see from Eq. 2 that a graph of $y_m$ vs. $m$ should be a straight line with a slope $\lambda D/d$. If this isn’t clear, imagine replacing the symbols in Eq. 2 with the generic symbols for the equation of a line:

$$y_m = \left(\frac{\lambda D}{d}\right) m + 0$$

$$y = \text{(slope)} \times + b$$

• Therefore, the strategy is to: i) graph $y_m$ vs. $m$, which should be linear; ii) find the slope of the best fit line, and then iii) use the slope and known values of $d$ and $D$ to calculate the wavelength.

• Use Graphical Analysis to plot and fit the data with the function $y = Ax$, in which $A$, the lone fit parameter, is the slope. Record the slope below with its uncertainty and proper units, and print the graph so you can attach a copy to the lab report.
slope from best fit = _________________ ± _________________

**Result**

- Calculate the wavelength from the best fit slope. Show all your work, and report a value in nanometers (nm) to four significant figures. (Further rounding may be appropriate, depending on the results of the uncertainty analysis below.)

\[ \lambda = \text{_______________ nm} \]

**Uncertainty analysis**

- Contributions to the statistical uncertainty in the measured wavelength are made by uncertainties in all the relevant parameters; namely, i) uncertainty in the slit separation \(d\), ii) uncertainty in the slits-to-screen distance \(D\), and iii) uncertainty in the slope, due to statistical deviations of the \(y_n\)’s from a linear trend.

The uncertainty in the slit separation, since the manufacturer reports a nominal spacing to two significant figures of 0.25 mm, will be taken to be ±0.005 mm. The uncertainty in \(D\) was estimated back on page 3. The uncertainty in the slope, obtained from the *Graphical Analysis* fitting algorithm, was reported above.

Calculate the *fractional* uncertainty, to two significant figures, for each of these below. The first has been completed for you. Use it as a template. *(The percent uncertainty is obtained by multiplying by 100.)* **Circle the largest source of uncertainty.**

\[
\frac{\delta d}{d} = \frac{0.005 \text{ mm}}{0.25 \text{ mm}} = 0.020 \quad \frac{\delta D}{D} = \quad \frac{\delta \text{slope}}{\text{slope}} =
\]

The fractional uncertainty in the wavelength, due to these contributions, is the square root of the sum of their squares. Report your answer below to two significant figures. *(Show your work.)*

\[ \frac{\delta \lambda}{\lambda} = \]

The final step is to calculate the *absolute* uncertainty in the wavelength. It is the product of the wavelength and its fractional uncertainty. Show your work below and round the final result to one significant figure.

\[ \delta \lambda = \lambda \left( \frac{\delta \lambda}{\lambda} \right) = \]

**Result with uncertainty**

- Report your final result for the wavelength in nm. A good rule of thumb for sig figs is this: the least significant digit reported in the wavelength should hold the same place as the lone significant figure of its uncertainty.

\[ \lambda = \text{_______________ ± _________________ nm} \]
Great! You have just experimentally determined the wavelength of a laser. Before proceeding, check your result. If you haven’t noticed yet, the laser’s nominal wavelength, as reported by the manufacturer, should be printed on the laser casing. Compare this to your result. If you feel like your result is reasonable, move on. If your result seems “way off,” there might be something wrong with your work. Review your data and calculations or consult with the instructor.

Experimental $\lambda = \phantom{00}0.0000\phantom{00}$ Manufacturer $\lambda = \phantom{00}0.0000\phantom{00}$

2. **Single Slit Diffraction**

**Introduction**

In Part 1 you determined the laser’s wavelength by analyzing the two-slit interference pattern, but this was only possible because you were given the manufacturer’s reported value for the slit width. In Part 2 you will see how this process can be reversed. Now that you know the laser’s wavelength you can analyze an interference pattern to determine the size of the aperture or obstruction that the light encounters.

In this experiment, instead of two slits, the light will encounter a single narrow slit. The geometry of the experiment is shown in Figure 5. The so-called single slit diffraction pattern will be observed on the screen. The exact nature of the pattern depends on both the light wavelength $\lambda$ and the slit width $a$. The goal of this activity is to determine the slit width.

**Activity #2 Goal: Determine the width of a narrow slit.**

![Figure 5: Double slit interference geometry](image)

**Procedure**

- Replace the Multiple Slit Set with the Single Slit Set. Turn the wheel on the Single Slit Set so as to center the single slit having a nominal slit width of $a = 0.04$ mm.

- Secure the Single Slit Set so that the slits-to-view screen distance $D$ is at least 1000 mm. As before, a precise measurement of this distance is important; make it with care, and record $D$ in the space below.

  $D = \phantom{00}0.0000\phantom{00}$ ± $\phantom{00}0.0000\phantom{00}$ mm

- Turn on the laser and make any needed final adjustments to obtain a nice bright and horizontal interference pattern.

- As before, attach a blank piece of masking tape across the view screen and diffraction pattern.
• The diffraction pattern on the view screen should be a horizontal line of alternating bright and dark spots something like that shown in Figure 6 below.

![Figure 6: Single slit diffraction pattern, showing locations of two minima.](image)

• Notice that the maxima are much broader in this pattern than in the double slit pattern from Part 1. In fact, because they are so broad, marking the positions of the maxima with high precision is imprecise. To analyze this pattern with precision it is better to mark the minima. Do so. With a pencil mark all the minima that appear on the view screen to the left and right of center. You should be able to mark at least three and probably four on each side. (Note that the minima, especially the first, might not appear perfectly dark. Mark any point where the intensity drops significantly and then rises again. Consult your instructor if you are having trouble.)

• When finished, remove the tape from the screen and, without wrinkling it, attach it in the box below.

![Tape tape here.](image)

• Use the Vernier calipers to measure the distance \( \Delta y \) between the symmetric pairs of minima on either side of the central maximum as shown in Figure 7. We’ll label the minima with an integer index \( m = \pm 1, \pm 2, \pm 3, \ldots \). (The center of the screen is bright, so there is no \( m = 0 \) minimum.)

![Figure 7: Calipers shown measuring the distance between \( m = \pm 2 \) minima](image)
• Record your measurements in the table below, and compute the $y_m$ values.

<table>
<thead>
<tr>
<th>$m$</th>
<th>$\Delta y_m$ (mm)</th>
<th>$y_m = \frac{\Delta y_m}{2}$ (mm)</th>
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<tbody>
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• Now that the minima have been located, look carefully at how the intensity of the maxima change as you move away from the center of the view screen. Make a sketch of intensity vs. position on the graph below for half of this symmetric pattern. As before we’ll say the intensity at the center of the screen, the brightest point, is 100.

When finished, mark and label two points on the graph where DESTRUCTIVE interference is occurring.

For a slit width $a$, it can be shown that these points of destructive interference occur at angles which satisfy the condition,

$$a \sin \theta_m = m \lambda \quad m = \pm 1, \pm 2, \pm 3, \ldots$$

(3)

And, by invoking the same small angle approximation that was described on page 3, Eq. 3 can be recast as an expression for the locations of the minima as measured along the view screen with a ruler or Vernier calipers. With $\sin \theta \approx y/D$, we have

$$y_m = \frac{m \lambda D}{a} \quad m = \pm 1, \pm 2, \pm 3, \ldots$$

(4)

• The goal now is to determine experimentally the width $a$ of the single slit, which can then be compared to the nominal slit width ($0.04$ mm) that is printed on the face of the Single Slit Set. You might say we are asking, “Is the slit width actually what the manufacturer says it is?” To begin the analysis, note that the form of Eq. 4 shows that a graph of $y_m$ vs. $m$ should be a straight line with a slope of $\lambda D/a$. 

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Therefore, the strategy is to: i) graph $y_m$ vs. $m$, which should be linear, ii) find the slope of the best fit line, and then iii) use the slope and your experimentally determined value of $\lambda$ to calculate the slit width.

- Use *Graphical Analysis* to plot and fit the data with the fit function $y = Ax$, where $A$, the lone fit parameter, is the slope. Record the slope below with its uncertainty and proper units, and print the graph so you can attach a copy to the lab report.

  \[
  \text{slope from best fit} = \text{____________________________ } \pm \text{____________________________}
  \]

**Result**

- Calculate the slit width from the slope and experimentally determined wavelength from part 1 of the lab. Show all your work, and report a value in millimeters (mm) to *four* significant figures. (Further rounding may be appropriate, depending on the results of the uncertainty analysis below.)

  \[
  a = \text{____________________________ mm}
  \]

**Uncertainty analysis**

- Contributions to the statistical uncertainty in the measured wavelength are made by uncertainties in all the relevant parameters; namely, i) uncertainty in the wavelength $\lambda$, ii) uncertainty in the slit-to-screen distance $D$, and iii) uncertainty in the slope, due to statistical deviations of the $y_m$’s from a linear trend.

  The uncertainty in the wavelength was determined in Part 1; use that value. The uncertainty in $D$ was estimated on page 8. The uncertainty in the slope, obtained from the *Graphical Analysis* fitting algorithm, was reported above.

  Calculate the *fractional* uncertainty, to two significant figures, for each of these below. (The *percent* uncertainty is obtained by multiplying by 100.) *Circle the largest source of uncertainty.*

  \[
  \frac{\delta\lambda}{\lambda} = \quad \frac{\delta D}{D} = \quad \frac{\delta\text{slope}}{\text{slope}} =
  \]

  The fractional uncertainty in the slit width, due to these contributions, is the square root of the sum of their squares. Report your answer below to two significant figures. (Show your work.)

  \[
  \frac{\delta a}{a} =
  \]

  The final step is to calculate the *absolute* uncertainty in the slit width. It is the product of the slit width and its fractional uncertainty. Show your work below and round the final result to *one* significant figure.

  \[
  \delta a = a \left( \frac{\delta a}{a} \right) =
  \]

  - Report your final result for the slit width in mm. A good rule of thumb for sig figs is this: the least significant digit reported in the slit width should hold the same place as the lone significant figure of its uncertainty.

  \[
  a = \text{____________________________ } \pm \text{____________________________ mm}
  \]
Great! You have just experimentally determined the slit width using a laser. Before proceeding, check your result. The slit width has a nominal width of 0.04 mm, as marked by the manufacturer on the Single Slit apparatus. Compare this to your result. If you feel like your result is reasonable, move on. If your result seems “way off,” there might be something wrong with your work. Review your data and calculations or consult with the instructor.

Experimental $a = \underline{}$       Manufacturer $a = \underline{}$

**Follow-up**

We’ve spent all of Part 2 analyzing diffraction from one particular narrow slit. Now observe how a diffraction pattern depends on slit width and shape. Rotate the slit selector wheel to the different slit widths and shapes and observe the patterns. Try out the “variable width slit” on the selector, and slide the wheel through the smoothly varying slit width while observing the pattern.

What happens to the pattern as the slit width changes from large to small? Does the pattern become more or less “spread out?” (Remember this result. It is an essential feature of diffraction.)

### 3. Diffraction by an obstruction

**Experiment**

Design an experiment to measure the diameter of a strand of your hair (or a thin wire). Scissors are available. ☃️ Diffraction from a thin obstruction like a strand of hair, which can be thought of as the negative of a narrow slit, is very similar to the diffraction pattern you observed in Part 2. In fact, the minima in the diffraction pattern on a distant view screen occur at angles that satisfy Eq. 3.

**Activity #3 Goal: Determine the width of a thin wire or strand of hair.**

Carry out the experiment, draw a diagram of your setup, record your measurements, show your analysis and calculations and state your conclusion. Compare your result with other classmates.