Electromagnets

Historical Note

The year 1820 was an incredibly important year in the history of magnetism. In April the Danish physicist Hans Christian Ørsted (1777-1851) reported on an experiment he had done in 1819 in which a compass needle rotated when brought near a wire that was carrying electric current. His report reached Paris in September. Within a week André-Marie Ampère (1775-1836) had repeated Ørsted’s experiment and had extended the result, showing that two current-carrying wires could attract or repel each other, depending on the relative directions of the currents. Within a few months the French physicists Jean-Baptiste Biot (1774-1872) and Félix Savart (1791-1841) developed a mathematical formula for the field produced by a tiny length of wire that was similar to Coulomb’s law for electric charges. Before the year was out François Jean Dominique Arago (1786-1853) is said to have produced the first true electromagnet—a coil of wire into which an iron core could be placed to increase the magnetic strength. In this lab you will explore the magnetic fields produced by electric currents.

In 1821 the Englishman Michael Faraday (1791-1867), with the help of Humphrey Davy and William Wallaston, developed a crude electric motor. Faraday later developed the concept of a magnetic field to describe the effect a magnet has on materials some distance away from it. In 1831 Faraday and the American Joseph Henry (1797-1878) independently discovered electromagnetic induction, which we will explore in the next lab.

Goals

By the end of this lab you should be able to answer these questions

- What magnetic field does an electric current produce?
- How can superposition be used to predict magnetic field strength?
- How can you produce a uniform magnetic field?

1. The field due to an infinitesimal length of wire

Introduction

Biot and Savart found that the magnetic field due to an infinitesimally-short piece of wire carrying a current behaved somewhat like the electric field due to a charge. That is, it depended on the distance according to $1/r^2$. But, the direction of the field and the way the field depended on the relative directions of the current and displacement vector, were totally different.

In modern notation

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{l} \times \hat{r}}{r^2}$$
In this equation $i$ is the current in the tiny segment of wire of length $dl$. The length $r$ is the distance from the current to the point where the field is being measured. The vector $\hat{r}$ is a unit vector that points from the current to the place that the magnetic field $dB$ is measured. Finally, $\mu_0/4\pi$ is a constant.

That’s a pretty imposing equation, but the vectors and the cross product can be reduced to a right-hand rule: If the first finger of your right hand points in the direction of the current-carrying wire, $dl$, and your palm or second finger in the direction of $r$, then your thumb points in the direction of the magnetic field, $B$.

**Procedure**

Explore how the magnetic field from a short length of wire depends on direction and distance.

- There is a folder called *Magnetism* on the computer’s desktop that contains a number of programs to be used in this lab. Open it and run the program *Biot*.

- First, choose the direction of the wire by clicking on either the blue or green button at the top of the screen. You will see a wire made-up of six short segments. One segment, labeled $dl$, is colored blue to indicate that there is current in that segment.

- Put the cursor on the end of the red $r$ arrow, press the left mouse button, and drag the point. Watch how the length of the green $dB$ vector changes and note its direction. If you are now looking at the wire in the plane, choose the other button and see the field from a wire perpendicular to the plane.

  (Hints on using the program—click right button and drag mouse to change viewpoint, click both buttons and drag mouse to zoom in and out.)

**Summarize your work:**

1. In what direction or directions do you point $r$ with respect to $dl$ to have a zero magnetic field?

2. What’s the relative angle between $dl$ and $r$ that gives the largest magnetic field?

3. Does the right-hand rule work? Explain your answer.

4. How does the strength of the magnetic field depend on the distance from the current? (Hint: record the value of $r$ and $dB$ at one distance, then double the size of $r$ and find the ratio of the new to the old magnetic field. Be sure that you don’t change the angle between $dl$ and $r$.)
2. The magnetic field due to a long rod

Introduction
Of course, in the real world we don’t find infinitesimal lengths of wire. But we can think of a real wire, whether straight or bent, as a series of tiny wire segments strung together. As the lab proceeds we will explore the field produced by straight wire and wire bent into a loop and then a series of loops. We begin with a long straight wire, or rod.

Procedure
We can think of a long thin rod as a series of tiny wire segments all in the same direction. When charge flows in the rod, this electric current produces a magnetic field.

- Run the program B_Wire.
- Each click of the left mouse button calculates the field due to a different small segment of the wire at a point in space near the rod. Notice how the fields add together. (Hint: Right click and drag to change the orientation for a better view.)
- Keep clicking to see how the fields from each segment add together at different locations. (Hint: Hold down the left button to speed things up.)
- When the program B_Wire has found the field at four locations near the rod, close the window and start the program B_Wire_All. Use the right mouse button to examine the field at various locations.

Summarize your work:
1. Does the right-hand rule hold? Point the thumb of your right hand in the direction of the current. The curl of your fingers gives the sense which the field circles around the wire. Is this rule consistent with the field vectors generated by the program?

2. Do the fields from all the tiny wire segments make equal contributions to the total magnetic field? Explain.

3. Does the field depend on the location along the rod? On the distance from the rod? Explain.

3. From rod to loop

Introduction
In this section you will predict how the magnetic field produced by a steady current in a rod changes when the rod is bent into a loop. As an aid in forming your hypothesis a comparison will be made to a similar electrostatics system with which you are already familiar.

Prediction
Two charged objects are shown in Figure 2 below. One is a long thin straight rod having a uniform positive linear charge density. The second is a thin positively charge ring with the exact same linear charge density. (The objects are not interacting with each other;
consider them separately.) Both charged objects produce an electric field everywhere, but we will be considering the field only at a single point, marked \( P \) in each case. The points \( P \) are the same distance, \( r \), from the points marked \( x \) on each object. You should consider the ring as the rod, cut to the appropriate length, held fixed at point \( x \), and wrapped into a circle about point \( P \). This is useful because we want to consider how the field at point \( P \) changes as the rod is wrapped into a circle as just described.

\[ \begin{align*}
\text{Figure 2: A charged rod and ring} \\
\end{align*} \]

In each case the electric field at \( P \) can be thought of as the sum of contributions from each segment of the corresponding object. In the diagrams, four representative segments, of equal length, on the rod and ring, have been highlighted in gray and are labeled \( a-h \).

For each object, illustrate the contribution of each highlighted segment to the net field at \( P \). To do so, if the field from a segment happens to be in the plane of the page draw an arrow with its tail attached to point \( P \); if the field happens to be directed into or out of the page use the symbols \( \bigcirc \) or \( \bigotimes \), respectively. You should attempt to depict both the relative magnitude and direction of the field in each case. Symbolically, label these fields \( \vec{E}_a, \vec{E}_b, \ldots, \vec{E}_h \).

In the same way, depict the net field at point \( P \) for each case. Keep in mind that the net field is the superposition of contributions from all segments, not just the four representative ones. Symbolically, label these fields \( \vec{E}_{\text{net,rod}} \) and \( \vec{E}_{\text{net,ring}} \).

**Check your thinking:**

Your additions to the diagram should unambiguously illustrate the direction of each field vector. Whether you were successful in depicting their relative magnitudes or not, make your intention clear by ranking all ten vectors from greatest to least in magnitude. If any have the same magnitude, give them the same ranking. Write down your ranking in the box below.
Summarize your work:

1. Do you predict the net electric field at point $P$ for the ring to be greater, less or equal in magnitude to the field at point $P$ for the rod?

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___________________________________________________________________
___________________________________________________________________

2. In words, describe what you think is happening to the net field at $P$ when the rod is bent into a circle.

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Two neutral conductors are shown in Figure 3 below. (The objects are not interacting with each other; consider them separately.) One is a long thin straight rod in which there is a steady current. The second is a thin ring in which there is a steady current of the same magnitude. As a result of the current, both objects produce a magnetic field everywhere, but we will be considering the field only at a single point, marked $P$ in each case. The points $P$ are the same distance, $r$, from the points marked $x$ on each object. You should consider the ring as the rod, cut to the appropriate length, held fixed at point $x$, and wrapped into a circle about point $P$. This is useful because we want to consider how the field at point $P$ changes as the rod is wrapped into a circle as just described.

![Figure 3](image)

In each case the magnetic field at $P$ can be thought of as the sum of contributions from each segment of the corresponding object. In the diagrams, four representative segments, of equal length, on the rod and ring, have been highlighted in gray and are labeled $a$-$h$.

For each object, illustrate the contribution of each highlighted segment to the total field at $P$. To do so, if the field from a segment happens to be in the plane of the page draw an arrow with its tail attached to point $P$; if the field happens to be directed into or out of the page use the symbols $\Theta$ or $\circ$, respectively. You should attempt to depict both the relative magnitude and direction of the field in each case. Symbolically, label these fields $\vec{B}_a$, $\vec{B}_b$, ..., $\vec{B}_h$.

In the same way, depict the net field at point $P$ for each case. Keep in mind that the net field is the superposition of contributions from all segments, not just the four representative ones. Symbolically, label these fields $\vec{B}_{net,rod}$ and $\vec{B}_{net,ring}$. 
Check your thinking:

Your additions to the diagram should unambiguously illustrate the direction of each field vector. Whether you were successful in depicting their relative magnitudes or not, make your intention clear by ranking all ten vectors from greatest to least in magnitude. If any have the same magnitude, give them the same ranking. Write down your ranking in the box below.

Summarize your work.

3. Do you predict the net magnetic field at point \( P \) for the ring to be greater, less or equal in magnitude to the field at point \( P \) for the rod?

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4. In words, describe what you think is happening to the net field at \( P \) when the rod is bent into a circle.

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5. How is this case the same as the analogous electrostatics case? How is it different?

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4. **The magnetic field of a current loop in three dimensions**

**Introduction**

Representing the magnetic field of a current loop on paper is difficult, given its three-dimensional structure. In this section you will look at some computer generated three-dimensional visualizations of particular field vectors being constructed from the superposition of contributions from segments along the loop.

**Procedure**

- Start the program `B_Loop_1`.
- Click the left mouse button to see how the field at the loop’s center is formed by the summation of contributions from individual segments. (Program Hint: Click the right mouse button and drag to rotate the image.)
• Continue clicking to see the field at other locations in the plane of the loop other than
the center of the circle. Observe at least four more locations, including one outside
the loop. (Program Hint: Click on both mouse buttons and drag to zoom out so you
can see the length of all vectors.) In these examples, because different loop
segments are different distances from the location, the magnetic fields due to these
segments vary in size, but note how in all cases the magnetic field is parallel to the
central axis of the loop.

• Close B_Loop_1 and start the program B_Loop_2 to examine the field on the
central axis of the loop, but off the loop’s plane.

• Begin clicking the left mouse button to view the field being constructed at a point on
the central axis. Note that the field due to each segment is the same size, but in a
different direction. Still, because of the symmetry, they add to form a field along the
axis.

• Continue clicking to view the field being constructed at one more location.

**Summarize your work:**

1. Check the right-hand rule for individual current segments. Point the thumb of your
right hand in the direction of the current of the segment. The curl of your fingers
gives the sense which the field circles around the segment. Is this rule consistent
with the field vectors generated by the program for points both inside and outside the
loop? Explain

2. Is the field the same strength everywhere inside the loop? If not, where it is largest?
Smallest?

5. **Magnetic field of a helical turn**

Unfortunately, we can’t measure the field in the plane of a wire loop because of the
limitations of our magnetic field sensors. Instead, we’ll use a wire wound in a helix,
using the Slinky® toy. Start the program B_Slinky_1 to explore how the field from a
single helical turn differs from a loop. Click the left button to step through the
calculations.

Notice that the field isn’t quite along the axis and that when measuring the field near the
wire the fields from individual segments point in wildly different directions.
Nevertheless, the general pattern is close to that of the simple loop.
6. Measuring the field due to current in a long straight rod

Introduction

We will begin the measurement part of this lab by first measuring the field produced by current in a long straight rod. As you make the measurement, keep in mind what you have learned in the preceding activities about the direction and magnitude of such a field. At the same time, anticipate the upcoming activities when the field from a loop will be measured. How will the results change?

Preparation

- The magnetic field probe should be attached to a support rod as shown in the picture below. Check that the white dot is facing horizontally. (In other words, a line passing perpendicularly through the face of the white dot should be horizontal.)

![Figure 4: Apparatus for field measurement](image)

- Use a ruler to check the height of the center of the white dot. It should be 11.5 cm above the table top. Adjust it if necessary. (This height will position the probe at the vertical center of the Slinky®.)

- Slide the magnetic field probe through the slot in the PVC pipe so it is positioned at the horizontal center of the pipe. (View down the pipe while positioning it.) At the same time, make sure the blue support track is perpendicular to the longitudinal axis of the PVC pipe.

The goal in all these steps is to have the longitudinal axis of the Slinky® pass perpendicularly through the center of the white dot on the probe. Make it so.

- Check that the thick copper wire is parallel with the table top and resting across the top of the PVC pipe as shown in the picture. (The goal here is to have the copper wire directly above and parallel to the plastic shaft of the magnetic field probe. This will cause the field produced by current in the wire to be perpendicular to the face of the probe, ensuring that the total field is measured.)

- With an alligator clip at each end of the copper wire, connect it to the power supply. Note which end is positive so you can tell the direction of current.

- Prepare the power supply to act as a constant current source (rather than a constant voltage source.)
  - Set the digital display to read Amps using the black switch to the right of the display.
  - Turn all four control knobs fully counterclockwise and then turn on the supply. It should read 0.00 A.
  - Turn the fine voltage control knob all the way clockwise. The current should still read 0.00 A.
  - Adjust both the coarse and fine current control knobs, as needed, until the current reads 3.00 A. You should not have to adjust the supply again until the very end of the lab. It is ready to supply a constant current of 3.00 A.
  - Turn off the supply.
Measurement

- Start the program Logger Pro and open the file named Slinky.
- With the power supply off, click on the “zero” button. Now Earth’s magnetic field will be automatically subtracted from each of your measurements.
- Turn on the power supply and verify a 3.00 A current. In Logger Pro click collect. The graph should display a fairly constant value of magnetic field being produced by the current.
- Determine the strength of this field by averaging over many data points. To do this, left-click and drag over 5 to 10 seconds worth of data on the graph. Click the statistics button on the tool bar and record the mean field, \( B_{\text{rod}} \).

\[
B_{\text{rod}} = \underline{___________ \mu T} \quad d = \underline{0.037} \text{ m}
\]

(In this setup, the distance, \( d \), from the rod to the probe is the same as the radius of the Slinky®. It has been measured for you and is already recorded above.)
- Does the magnetic field follow the right-hand rule? Explain how you reached your conclusion.

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7. Measuring the field at the center of a current loop

Introduction

Now you will measure the field at the center of a current loop (actually one turn of a helix), and compare this result to the field from the straight rod. Note that this experimental comparison is similar to the theoretical comparison you made earlier. In particular, the apparatus has been configured so that the radius, \( R \), of the loop is the same as the distance, \( d \), from the rod to the probe in the measurement just made.

Procedure

- Remove the long copper wire rod so that it is out of the way. The Slinky® should be stretched enough so the spacing between windings is a little more than the width of the magnetic field probe.
- Check that the magnetic field probe is still positioned at the center of the Slinky® so that the longitudinal axis of the Slinky® passes perpendicularly through the white dot.
- Prepare to send current through exactly one turn of the helix by connecting the Slinky® to the power supply as shown in the picture. (Are you convinced current will appear in only a single turn?)
- Turn on the power supply and confirm that it is supplying 3.00 A.
- Measure the field at center of loop using the same averaging technique used in the last section. Record your result.

\[
B_{\text{loop}} = \underline{___________ \mu T} \quad R = \underline{0.037} \text{ m}
\]
**Conclusions**

Using your results from this and the last section calculate the ratio $B_{\text{loop}}/B_{\text{rod}}$ and record this result in the *Experiment* column in *Table 1*.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{\text{loop}}/B_{\text{rod}}$</td>
<td></td>
</tr>
</tbody>
</table>

*Table 1*

What is the theoretical expectation for this ratio? Determine it using the expressions for the $B$-field due to current in a long rod and at the center of a current loop. Show your work in the box and record this value in the *Theory* column of *Table 1*. (You might need to consult your textbook or friendly instructor.)

Calculate the percent error between your experimental value and the theoretical value.

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Was your prediction recorded in question #3 from section #3 correct? Comment.

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**8. From one loop to a solenoid**

**Introduction**

In this section you will examine how the magnetic field produced by a steady current in a single loop of wire changes when more loops are added.

**Prediction**

In the previous section, as an approximation to a loop, you measured the magnetic field at the center of a single turn of a helix due to a steady current flowing through it. Consider how you would expect the B-field to change as the number of loops increases while maintaining the same current. Let’s agree to increase the number of loops symmetrically by adding the same number of loops to both the left and right of the original single loop. For instance, the first case would be to consider what happens when one loop is added to the left and the right. (Practically, this will be achieved by simply moving the two alligator clips, one turn to the left and one turn to the right, respectively.) This would create a three loop system with the magnetic field sensor at the very center. The next case would be a five loop system, created by adding another loop to the left and right.

Before making the measurement, make a prediction. Circle the one graph from the selection below that you think accurately depicts the pattern that would develop if you plotted magnetic field versus number of loops.
Check your thinking:

1. Which of the following is consistent with your thinking and your choice? If none of these, add your own. (If you haven’t already, carefully consider the direction of current flow in each loop and the direction of the resulting $B$-field contributions from each loop.)

   A. I predict that the added loops will tend to increase the magnetic field over and above its original single-loop value.
   B. I predict that the added loops, because they are on both sides, will tend to cancel the original single loop magnetic field.
   C. I predict that the magnetic field will remain constant because the current has not changed.
   D. Other…

2. Answer the following question in a way that is consistent with your choices above. By tripling the number of loops, for the same current, the magnetic field will

   A. triple
   B. more than triple
   C. increase, but less than triple
   D. remain constant because the current has not changed
   E. drop to zero because of complete cancellation
   F. be reduced, but the cancellation won’t be complete

3. In words, provide a reasonable explanation for your prediction.

   __________________________________________________________
   __________________________________________________________

**Experiment**

Conduct an experiment to test your prediction. Make sure the current remains constant, that you do not move the magnetic field sensor from its central position, and that you do not significantly disturb the slinky as you reposition the alligator clips. You should use the averaging technique for any field measurements. Record your data in the table below and plot them using *Graphical Analysis*. (Print and attach the graph.)
### Electromagnets

<table>
<thead>
<tr>
<th>Number of Loops</th>
<th>Magnetic Field (μT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
</tr>
<tr>
<td>All</td>
<td></td>
</tr>
</tbody>
</table>

*Table 2*

**Summarize your work:**

1. In words describe how the axial field depends on the number of loops for a fixed current and spacing between turns.

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2. Provide a reasonable explanation for this result.

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3. Was your prediction correct?

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### 9. The solenoid: comparing experiment and theory

**Introduction**

According to Ampere’s law, the field inside a solenoid is given by $B = \mu_0 n i$, where $n$ is the turns density, or number of turns per meter. In this section you will investigate this equation.
a. **The central longitudinal field**

**Procedure**

- Record the field you measured for the entire solenoid from the last row in Table 2.
  \[ B_{\text{solenoid experiment}} = \text{______ } \mu T \]
- Determine the winding density of your solenoid by measuring its length and counting the number of loops. Use these numbers to compute \( n \).
  \[ n = \left( \frac{\text{______ turns}}{\text{______ m}} \right) = \text{______ turns/m} \]
- Compute the field of the solenoid that is given by Ampere’s law
  \[ B_{\text{solenoid theory}} = \text{______ } \mu T. \]
- Compare your measured result with this theoretical value by calculating a percent difference.

______

______

In the following sections you will check some of the other predictions of the solenoid equation: \( B = \mu_0 n i \).

b. **Dependence of field on current**

**Procedure**

- Measure the central longitudinal field as a function of current. Have all windings of the solenoid in the circuit. Record your results in the Table 3.

<table>
<thead>
<tr>
<th>Current (A)</th>
<th>Magnetic Field (μT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+3.00</td>
<td></td>
</tr>
<tr>
<td>+1.50</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>-1.50</td>
<td></td>
</tr>
<tr>
<td>-3.00</td>
<td></td>
</tr>
</tbody>
</table>

*Table 3*

- Does the field depend linearly on the current? Explain.

______

______
c. Dependence of field on winding density

**Procedure**
- Explore qualitatively how the central longitudinal field depends on winding density. Return the current to 3.00 A. Begin collecting data with Logger Pro. While it is collecting, slowly stretch the slinky and observe the change, if any, in the field strength.
- Describe what you observed and if it is consistent with the solenoid equation, \( B = \mu_0 ni \).

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d. The off-axis interior field

**Introduction**
The solenoid equation, \( B = \mu_0 ni \), is valid on the longitudinal axis, but not only on the longitudinal axis. For an ideal infinitely-long solenoid it gives the correct field everywhere in the solenoid’s interior. In other words, inside an ideal solenoid, the magnetic field is uniform! Is the field inside your solenoid uniform?

**Procedure**
- Check that all windings of the solenoid are in the circuit and the current is set at 3.00 A.
- Starting on the longitudinal axis, record the field as you pull the magnetic field probe off-axis, one centimeter at a time. As usual, use the averaging technique.

<table>
<thead>
<tr>
<th>Distance from Central Axis (cm)</th>
<th>Magnetic Field ((\mu)T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td></td>
</tr>
</tbody>
</table>

*Table 4*

**An application**
What are the uses for the uniform magnetic field inside a solenoid? One important use is in MRI, magnetic resonance imaging.

Solenoids used in MRI machines typically produce fields of 1.5 T or 3.0 T. The fields must be extremely uniform, differing by less than a microtesla, over the region of the body to be imaged. (This is a variation of less than 0.0001%.) The wire used is superconducting so that it has no resistance. Therefore there is no voltage across the solenoid, and no power dissipated by it.
How uniform was the field inside your solenoid? That is, if you wanted to do an experiment in a field that did not change by more than 10%, how far away from the center of the solenoid could you construct your experimental apparatus?

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**e. The exterior field**

**Procedure**

- For an infinitely long solenoid the field outside is extremely small. Your solenoid is hardly infinitely long. Measure the field outside the Slinky®.

<table>
<thead>
<tr>
<th>Distance from Central Axis (cm)</th>
<th>Magnetic Field (μT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td></td>
</tr>
<tr>
<td>5.0</td>
<td></td>
</tr>
<tr>
<td>6.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 5

- To understand what’s happening, start the program B_Slinky. (Hint: If a warning bar appears, click it, and choose to allow blocked content.) Start with one loop. Then add additional loops, watching the strength of the field inside and outside the loops. (Hint: Left click in the picture and a numerical value of the field will appear at the lower left.) What happens to the interior and exterior fields as the solenoid gets longer?