RC Circuits

Introduction
In this lab you will explore RC circuits.

1. Designing a time delay circuit

Introduction
You will begin your exploration of RC circuits in the context of an actual circuit design problem. In designing a circuit to meet certain specifications you will learn the important features of RC circuits.

Mission
Your mission is to build part of the circuitry for a home alarm system. When the front door of the house is opened from the outside, possibly due to a burglar, the alarm circuitry is triggered, but the actual audible alarm does not sound immediately. Instead, there is a built-in 10 second delay, allowing the owner of the home to enter the house and deactivate the alarm by entering a code on a control panel just inside the doorway.

You will be building the circuit that creates a 10 ± 1 second delay. A block diagram of the circuit is shown below.

![Block Diagram]

$V_{\text{set}}$ is a fixed voltage, it will be 4 V in our case, that is applied to one input of something called a comparator. $V_{\text{sense}}$ is a voltage that changes with time, and is applied to the other input of the comparator. The comparator, surprise-surprise, is a chip that compares $V_{\text{set}}$ and $V_{\text{sense}}$. If $V_{\text{sense}}$ is greater than $V_{\text{set}}$ the comparator's output is such that the alarm sounds. If $V_{\text{sense}}$ is less than $V_{\text{set}}$ the comparator's output is such that the alarm is silent.
The basic idea is to keep $V_{\text{sense}}$ less than $V_{\text{set}}$ for a full 10 seconds after the door opens, at which time $V_{\text{sense}}$ becomes greater than $V_{\text{set}}$ and sounds the alarm. You have two circuits to design.

**Voltage Divider**

*Circuit 1*: $V_{\text{set}}$ for your circuit will be 4 V. You have to design a circuit that will apply 4 V to the $V_{\text{set}}$ input of the comparator. At first blush this might seem easy, just connect the power supply to the $V_{\text{set}}$ input and set the supply to 4 V. That would work, but it is not allowed. (I make the rules.) The only voltage available to you is 10 V. In other words, you may power your circuit with the dc supply at your table, but the supply must be set at 10 V. You need a way to apply a fraction of the full supply voltage, 2/5 of 10 V or 4 V, to the $V_{\text{set}}$ input.

This is a very common situation. In electronic devices that require dc voltages, the power supply typically provides only one or a few fixed voltage levels. There may be many parts of the circuitry that require voltage levels other than those few supplied. In your case, how do you get 4 V from the only available voltage level, 10 V?

This can be accomplished with two resistors in a combination called a voltage divider. As shown below, it is nothing more than two resistors in series. (We will call the negative, low potential, terminal of the power supply, circuit ground. When we refer to the voltage at some point in the circuit, like 4 V at the output, what is really meant is the potential difference between that point and circuit ground.)

![Diagram of Voltage Divider](image)

Start designing (on paper, don’t build yet). Figure out two resistor values that will put 4 V at the output. Show your work below. In other words, justify your choices with calculations.
Now that you’ve done all that work, here are two constraints that might cause you to change your choices.

- The available resistors come in a limited set of stock values. The lab should have any value in the table of values that you should find on your desk. (You are allowed to use series and parallel combinations, but don’t get carried away. A good design will limit the part count to two or three resistors.)

- The lab’s resistors can dissipate no more than ¼ Watt. Don’t exceed this limit. Calculate the power dissipation in each of your resistors and show that it is indeed less that 0.25 W. Show your work.

The design board

You are about to build your divider. You will build it on the design-board at your table. An empty design-board is shown below in Figure 3(a). Components can be inserted directly into the sockets on the board.

To distribute power all the sockets down the left column are connected together and will be connected to 10 V. All the sockets down the right column are connected together and will be connected to circuit ground. The sockets in the two wide middle sections are connected horizontally, not vertically, and are used to place components. But the five sockets in a row on the left are not connected internally to the adjacent set of five sockets on the right. Figure 3(b) shows these hidden connections.

A number of colored jumper wires have been pre-inserted in the board. Do not remove these. Four metal “posts” have also been inserted. These will be used to easily connect cables with alligator clips. Do not remove them either.

Place the resistors of your voltage divider as shown in Figure 3(c). The high potential side of the divider (point X in the Figure 2 diagram) must be inserted in row 1. The low potential side of the divider (point Z in the Figure 2 diagram) must be inserted in row 11. The point connecting the “top” and “bottom” resistors of the divider (point Y in the Figure 2 diagram) must be connected to row 6.

Figure 3(c) shows how a divider made of two physical resistors should be connected. Even if your divider is made of more than two resistors, the same connection instructions must be followed.
Divider build and test  Go ahead and construct your voltage divider. Test it too! Set the supply at 10 V and connect it to your design-board as shown in Figure 3(c). With your multimeter measure the voltage across the supply terminals, \( V_{\text{supply}} \), and measure the voltage across the bottom half of the divider, \( V_{\text{set}} \). If \( V_{\text{supply}} \) isn’t 10 V, adjust the supply until it is. If \( V_{\text{set}} \) isn’t close to 4 V you’ll need to modify your resistor values. (If anything is smoking, turn off the supply.) Describe any modifications, if any, you had to make.

\[
V_{\text{supply}} = \text{__________}
\]

\[
V_{\text{set}} = \text{__________}
\]

Great! Now for the second circuit.

Safety  SAFETY FIRST!!! WEAR YOUR SAFETY GLASSES!!! If you construct the second circuit incorrectly there is a chance that a capacitor could explode in your face. This would not be a fiery explosion. Instead, through an electrochemical reaction inside the capacitor, a gas could evolve, pressure would build up and the capacitor casing would burst open. It would be much like a popcorn kernel popping, but more violent. Shards of capacitor casing or guts could be projected at high speed into your eyeball(s). OUCH! This will not happen if your circuit is connected correctly. ASK YOUR INSTRUCTOR TO EXAMINE YOUR CIRCUIT BEFORE YOU TURN ON THE POWER SUPPLY. EVEN BETTER, LEARN HOW TO CONNECT THE CAPACITOR CORRECTLY, THEN DO SO.

RC circuit  Circuit 2: The output of the second circuit is \( V_{\text{sense}} \), which will be applied to the other input of the comparator. \( V_{\text{sense}} \) changes with time. Your goal is to build a circuit so that \( V_{\text{sense}} \) starts at 0 V and then begins to increase when the door is opened, surpassing \( V_{\text{set}} \), nominally 4 V, after 10 seconds have elapsed, at which point the alarm sounds. This can be accomplished with an RC circuit like the one shown below in Figure 4. When the “door”, a switch, is closed the capacitor is shorted-out, and will be in a fully discharged state.
state, \( V_{\text{sense}} = 0 \text{ V} \). When the “door” is opened, the capacitor begins to charge. As before, 10 V is the only supply voltage available, so the battery is labeled 10 V.

![Figure 4]

**Charging curve** You should recognize that \( V_{\text{sense}}(t) \), the potential at point Y with respect to circuit ground, is just the voltage across the capacitor. It charges according to the classic RC circuit charging curve, \( V_{\text{sense}}(t) = V_{\text{max}} \left(1 - e^{-t/RC}\right) \). The shape of this function is shown on the graph below.

![Graph of the charging curve]
Add the following details to the graph:

A. Carefully add a scale to the vertical axis. Use a ruler to create a uniform set of tick marks, with values labeled. (This is not arbitrary, there is only one correct scale.)

B. There is one point on the horizontal time axis that corresponds to 10 seconds of elapsed time since the door was opened at $t = 0$ s. Mark the 10 s point on the time axis. (This is not arbitrary either. There is only one correct answer.)

C. Now that two points (0 s and 10 s) are marked on the horizontal axis, carefully complete the time scale along the entire horizontal axis by using a rule to create a uniform set of tick marks with labels.

D. Draw a vertical line segment from the 10 s mark on the time axis until it intersects the charging curve. Then draw a horizontal line segment from that intersection point to the vertical axis. Highlight the voltage on the vertical scale at the point of intersection. (There is only one correct answer. Make sure it is right.)

**RC Design**

Great! Now you have graphically illustrated the information needed to design your $RC$ circuit. The word *design* means finding values of $R$ and $C$ that will meet the circuit specifications. Do so. Justify and explain your choices. Show all your calculations in the space below.

Remember that there is a limited set of stock values for $R$ and $C$. If you haven’t taken that into account in your design choices, do so now. Use only a single capacitor. (I make the rules.) You can use more than one resistor in a series and/or parallel combination, but try to limit yourself to 2 or 3 resistors.

List the actual component values you plan to use, and if using a combination of resistors, describe the combination and its equivalent resistance.

\[
C = \text{______________}
\]

\[
R = \text{______________}
\]
The time constant

Depending on the values of \( R \) and \( C \), capacitors in an \( RC \) circuit charge more or less quickly. The rapidity with which the capacitor charges is characterized by the circuit’s time constant, \( \tau \). It is the product of \( R \) and \( C \). What is the time constant for your design? Show your work.

Mark and label the time corresponding to one time constant on the time axis of the charging curve above. Then draw a vertical line segment from this mark on the time axis until it intersects the charging curve. Then draw a horizontal line segment from that intersection point to the vertical axis. Label the voltage on the vertical scale at the point of intersection and record it here.

\[
V_{\text{sense}}(t = RC) = \text{__________}
\]

Check if this is correct. From the theoretical charging function calculate \( V_{\text{sense}}(t = RC) \). Show all your work. Is your theoretical result consistent with your graphical result?

It should be clear from your answers to the previous question that, although the time constant characterizes the rapidity with which the capacitor charges, it is not equal to the time it takes for the capacitor to fully charge. We will use a common rule-of-thumb. We will say that the capacitor is fully charged after five time constants have elapsed. Given this rule of thumb, how long will it take the capacitor in your design to fully charge? Show your work.

Mark and label the time corresponding to five time constants on the time axis of the charging curve above. Then draw a vertical line segment from this mark until it intersects the charging curve. Then draw a horizontal line segment from that intersection
point to the vertical axis. Label the voltage on the vertical scale at the point of intersection and record it here.

\[ V_{\text{sense}}(t = 5RC) = \] ________________

Check if this is correct. From the theoretical charging function calculate \( V_{\text{sense}}(t = 5RC) \).
Show all your work. Is your theoretical result consistent with your graphical result?

Safety

You are about to build your \( RC \) circuit. The capacitor must be inserted correctly or it could explode. Read the instructions carefully.

Go ahead and construct your \( RC \) circuit. You will be using the lower half of the design board for inserting these components. (Do not remove your voltage divider.) A design using a single resistor and capacitor would look like Figure 5(a). Note especially your capacitor. If it is the electrolytic variety, it will have labeling on its casing identifying one of the leads as “negative”. In a circuit, this lead must be maintained at a lower potential than the other lead or the capacitor could explode. In your circuit, the lead labeled negative must be inserted into a socket on the final row (row 23) of the design board. The other lead must be inserted in row 19. Note also that the resistance of your \( RC \) design must have its high potential side, point \( X \) in the diagram of Figure 4, inserted in row 13 of the design board. The low potential side of the resistance, point \( Y \) in the diagram of Figure 4, must be inserted in row 19 of the design board.

\[ \text{row 23} \]
\[ \text{row 19} \]
\[ \text{row 13} \]

As you prepare to test your circuit you need something to act as the “door”, shown schematically in Figure 4 as a switch that, when closed, shorts-out (discharges) the capacitor. A push-button switch will be used in the final testing of your circuit, when you connect it to the instructor’s comparator, but for now simply use a short length of wire.
As shown in Figure 5(b) insert the wire between rows 19 and 23 to “close the door” and short the capacitor. When you are ready to set the alarm time-delay circuit in motion “open the door” by removing the wire. Reinsert it when you want to reset the circuit.

**RC build and test**

In words explain the test you must perform to determine if your RC circuit works properly. Be specific. (The electrometer, familiar from previous labs, must be used to measure voltage.) Then, draw a schematic of the test circuit by adding instruments and connections to the diagram of the design board shown below.

Obtain your instructor’s initials before proceeding.

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**Explanation of test:**

The instructor has checked a) my explanation, b) that my capacitor is inserted properly, c) that my test circuit is correct, and d) that my electrometer is properly zeroed.
Test your circuit. Describe your results. If the time delay does not meet specs to your satisfaction, adjust the resistance to speed-it-up or slow-it-down. Then retest. Describe any redesign.

**Charging curve**

Next, you will measure your circuit’s charging curve and determine it’s time constant. Carefully measure the voltage across the capacitor as a function of time. Find a method that works best. I suggest marking the time every time an additional 2 V appears across the capacitor from 0 V to 10 V. (But you might also try marking the voltage for fixed time intervals.)

Create neat data tables in the space below for all your trials. Add explanatory notes if necessary.
Transfer your data to Graphical Analysis and plot the data. Fit the data with the built in function \( y = 10(1 - \exp(-x / A)) \). This is just the theoretical charging function with \( V_{\text{sense}} \rightarrow y \), \( t \rightarrow x \), \( V_{\text{max}} \rightarrow 10 \text{V} \), \( RC \rightarrow A \). Print the graph.

Record \( RC \) as obtained from the best fit of your data and compare to \( RC \) calculated from the component values currently in your circuit. What is the percent difference? (When contemplating the difference note that your resistors have a tolerance of ±5% and your capacitors a whopping ±20%)

\[
RC \text{ from best fit of charging data } = \quad \text{________________________}
\]

\[
RC \text{ calculated from component values } = \quad \text{________________________}
\]

\[
\% \text{ difference } = \quad \text{________________________}
\]

**Final test**

When you have finished this analysis and are satisfied with your circuit’s performance present it to the instructor for final testing. Obtain your instructor’s initials in one of the two boxes.

- [ ] The circuit’s time delay was 10 ± 1 s.
- [x] The circuit’s time delay was not within 1s of 10 s.

**Full Schematic**

Here is a schematic of the complete circuit. Add to the diagram values for the supply voltage, the resistances in your divider and the resistor and capacitor value in your \( RC \) circuit. Place the values next to the corresponding components.
Wrap-up

Check your thinking: The charging formula for the capacitor can be written as 
\[ V_c = V_{c,max} \left(1 - e^{-t/\tau}\right) \]. Four quantities, \( V_C, V_{C,max}, t \) and \( \tau \) appear.

As the capacitor charges which ones change with time? _________________
As the capacitor charges which ones remain constant? _________________

Although our attention was focused on the capacitor voltage, there are three other quantities changing with time as the capacitor charges. They are the charge on the capacitor \( q(t) \), the voltage across the resistor \( V_R(t) \) and the current \( i(t) \).

Sketch a graph of \( q(t) \). Explain your reasoning, calculate the maximum value of \( q \) in your \( RC \) circuit, and state the time at which it occurs.

Sketch a graph of \( V_R(t) \). Explain your reasoning, calculate the maximum value of \( V_R \) in your \( RC \) circuit, and state the time at which it occurs.

Sketch a graph of \( i(t) \). Explain your reasoning, calculate the maximum value of \( i \) in your \( RC \) circuit, and state the time at which it occurs.