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Customizing vacuum fluctuations for enhanced entanglement creation

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Abstract

This paper connects the creation of entanglement through cavity enhanced decay rate with practical design parameters such as cavity dimension and cavity mirror reflectivity. The clarification of specific physical parameters on cavity enhanced emission in relation to entanglement creation is discussed. It is found that entanglement increases as the size of the cavity decreases, or the reflectivity of the cavity mirrors increases. Additionally, the negative effect of individual qubit decoherence on the entanglement is discussed. These results can be used to design or choose a practical system for implementing entanglement between two qubits for quantum computation and information processing.

Keywords: quantum parameter estimation, open quantum systems and decoherence, quantum entanglement, quantum simulation

(Some figures may appear in colour only in the online journal)

Introduction

The creation of entanglement between two driven atoms through cavity enhanced decay rate is known to exist according to cavity QED [1–6]. Entanglement can also be created between two atoms by driving only a single atom [7]. The increase of entanglement is proportional to the collective decay rate of a cavity with two qubits. There is a broad spectrum of theoretical work on how to vary the collective decay rate γ between qubits and a cavity [8, 9] to create entanglement, however there is no published work on how the cavity dimensions or mirror reflectivity create entanglement. This article shows how the entanglement enhancement changes with cavity dimension and cavity loss to quantify the parameters of a practical experimental setup to observe the cavity enhanced entanglement. The current work finds that the entanglement increases as the size of the cavity is reduced or the reflectivity of the cavity mirrors increases. By understanding how the cavity dimensions and mirror reflectivity affect entanglement, a guideline can be created to choose resonant cavity properties needed to generate entanglement between qubits. The ability to generate entanglement between qubits is useful for future work in quantum computation and communication systems.

The model

The system consists of two identical two-level atoms (two qubits) coupled to a single-mode cavity which is continuously excited. In order to create a collective decay mechanism for the two atoms, they are placed in a high Q cavity that is resonant at the emission wavelength of the two atoms [10]. Thus in this system, each atom interacts with the laser light and undergoes Rabi oscillations as well as enhanced collective atom-cavity decay. The total angular momentum operators for the two qubits are defined as:

$$\hat{J}^{\pm} = \hat{\sigma}_1^{\pm} + \hat{\sigma}_2^{\pm}, \tag{1}$$

where the raising and lowering operators for each qubit are defined as $\hat{\sigma}^{+} = |g\rangle\langle e|$ and $\hat{\sigma}^{-} = |e\rangle\langle g|$.

In the $j = 1$ subspace there are three states

$$\begin{aligned} |1\rangle &= |e\rangle_1|e\rangle_2 \\ |2\rangle &= \frac{1}{\sqrt{2}}(|g\rangle_1|e\rangle_2 + |e\rangle_1|g\rangle_2) \\ |3\rangle &= |g\rangle_1|g\rangle_2. \end{aligned} \tag{2}$$

The fourth state, i.e. the $j = 0$ subspace, is

$$|\Psi_{j=0}\rangle = \frac{1}{\sqrt{2}}(|g\rangle_1|e\rangle_2 - |e\rangle_1|g\rangle_2). \tag{3}$$

The master equation describing the two-qubit atomic system driven by a laser with amplitude α is described by the density operator ρ defined as [11, 12]:

$$\dot{\rho} = -i\alpha[(J^{+} + J^{-}), \rho] + \Gamma_{\rm fr}\mathcal{D}[\sigma_1^{-}]\rho + \Gamma_{\rm fr}\mathcal{D}[\sigma_2^{-}]\rho + \Gamma_{\rm ca}\mathcal{D}[J^{-}]\rho, \quad (4)$$

where $\Gamma_{\rm fr}$ is the free space individual atom decay rate for both atoms, $\Gamma_{\rm ca}$ is the atom-cavity collective decay rate when the cavity is tuned to resonate at the emission wavelength λ, and $\mathcal{D}$ is a Lindblad superoperator defined as $\mathcal{D}[A]B \equiv ABA^{\dagger} - \{A^{\dagger}A, B\}/2$ for an irreversible evolution such as the free space and collective decay mentioned above [13].

In prior work [9] the relationship between the free space atom decay rate and the atom-cavity enhanced decay rate is not directly related to cavity parameters such as the cavity loss and size. The following equations (5)–(9) connect the atom-cavity enhancement to these parameters [14].

$$f = \frac{c}{\lambda}, \quad (5)$$

$$T = N\frac{\lambda}{c}, \quad (6)$$

$$Q = 2\pi f\frac{T}{k}, \quad (7)$$

$$l = N\frac{\lambda}{2}, \quad (8)$$

$$\frac{\Gamma_{\rm ca}}{\Gamma_{\rm fr}} = \frac{3Q}{4\pi^2}\left(\frac{\lambda}{l}\right)^3. \quad (9)$$

Here f is defined as the optical frequency of λ, c is the speed of light in a vacuum, Q is the optical cavity quality factor, N describes the integer number of half wavelengths $\lambda/2$, T is the round trip time for a photon in the cavity. By solving equation (9) the following relationship can be found. This result is one of the main findings of this paper.

$$\frac{\Gamma_{\rm ca}}{\Gamma_{\rm fr}} = \frac{12}{\pi}\frac{1}{kN^2}. \quad (10)$$

From equation (10) the ratio between the cavity enhanced decay rate $\Gamma_{\rm ca}$ and the free space decay rate $\Gamma_{\rm fr}$ is proportional to $1/N^2$ for a given cavity round trip loss k. Since cavity decay rate $\Gamma_{\rm ca}$ is only enhanced at cavity lengths of an integer multiple N of $\lambda/2$, this forces the enhancement ratio to only depend on the integer N. This implies that the physical cavity size to create a given amount of spontaneous emission scales directly with the emission wavelength of the atoms as $l = N\frac{\lambda}{2}$. For experimental implementation the scale of the cavity can be chosen by picking an emission wavelength that makes the dimensions of the cavity feasible to implement.

Since the mirrors only influence the electromagnetic field and the field decreases by the inverse square law, it is not surprising that the decrease in cavity decay rate by $1/N^2$ follows an inverse square law as well. Additionally as the size of the cavity is increased the cavity enhancement decreases to that of free space.

The ratio between the enhanced atom-cavity collective decay rate $\Gamma_{\rm ca}$ and the free space individual atom decay rate $\Gamma_{\rm fr}$ can be fine tuned using the cavity loss k and coarsely controlled by the cavity size N. For any given cavity loss k the maximum enhancement ratio is when $N = 1$. This indicates that the smaller the cavity the larger the cavity enhanced decay rate $\Gamma_{\rm ca}$. This relationship given by equation (10) is plotted in figure 1.

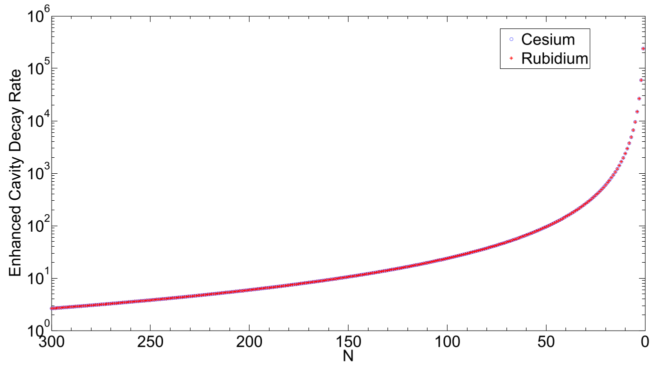

Figure 1. The blue circles and red asterisk represent the enhanced atom-cavity collective decay rate $\Gamma_{\rm ca}$ of a pair of cesium or rubidium atoms versus the size of the cavity in terms of N where each increment of N represents $1/2\ \lambda$. The cavity loss per round trip is 16 ppm.

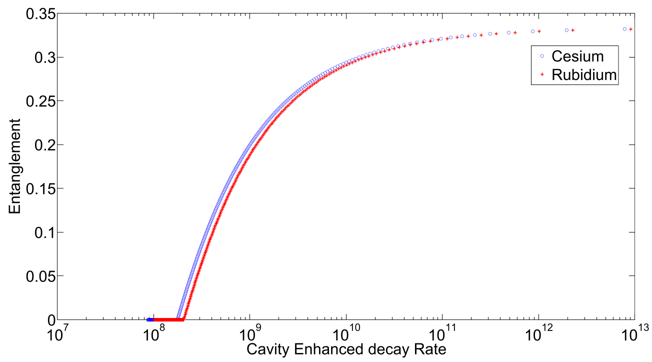

Figure 2. The blue circles and red asterisk represent the concurrence of a pair of cesium ($\Gamma_{\rm fr} = 3.282 \times 10^7$) or rubidium ($\Gamma_{\rm fr} = 3.811 \times 10^7$) atoms versus the enhanced atom-cavity collective decay rate $\Gamma_{\rm ca}$. The cavity loss per round trip is 16 ppm.

The entanglement measured by concurrence [15] can be calculated numerically [12] from the master equation steady-state solution in equation (4). The concurrence resulting from the numerical calculations for cesium and rubidium are shown in figure 2. These two atoms have different $\Gamma_{\rm fr}$ decay rates and emission wavelengths [16, 17]. The entanglement increases as the collective decay rate increases. The value for the cavity loss k was chosen based on equation (9) as well as the numerical results for entanglement for the maximum cavity enhancement size of $N = 1$. The round trip cavity loss $k = 2\pi N/Q = 16$ ppm gives the largest cavity loss that attains the best entanglement possible at $N = 1$. The two atoms can be differentiated since the free space decay rate of rubidium is more than that of cesium. The increase of free space decay requires more cavity enhanced decay rate to make up the difference to get the same level of entanglement. In figure 2 the horizontal axis corresponds to only the cavity decay rate and not the ratio. In figure 3 the bottom horizontal

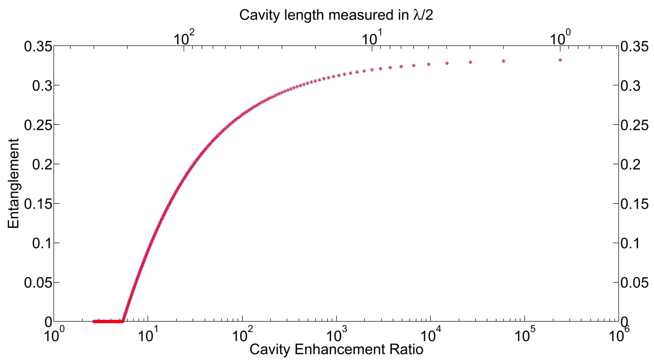

Figure 3. The blue circles and red asterisks represent the concurrence of a pair of either cesium (blue) or rubidium (red) atoms versus both the cavity length in terms of an integer number N half wavelengths on the upper horizontal axis, and the cavity enhanced emission ratio Γ_{ca}/Γ_{fr} on the lower horizontal axis. The wavelengths (λ) are normalized to the emission wavelength of the cesium or rubidium atoms. The cavity loss per round trip is 16 ppm.

axis corresponds to the ratio of atom-cavity decay rate to the free space decay rate. The abrupt transition from 0 entanglement to a smooth curve of increasing entanglement around the cavity enhancement ratio of 5.5 is similar to what has been found before [10]. The reason for the sharp transition is a subject for future research [18]. It becomes apparent from comparing the two plots that the amount of entanglement creation only depends on the size of the cavity with respect to the emission wavelength. The size of the cavity N as in the top horizontal axis of figure 3 changes the ratio between cavity enhanced decay rate Γ_{ca} and the free space decay rate Γ_{fr} of the atom. The interesting point is that the type of atom is not important for the entanglement generation. The atom scales the size of the cavity by the emission wavelength λ. This makes sense since without the cavity there is no entanglement generated between two atoms in free space. Since prior work was only concerned with arbitrary ratios between cavity enhancement and the free space decay rate this interesting property has been overlooked up to now.

Discussion

A number of papers have discussed the counter-intuitive method of increasing entanglement by arbitrarily increasing the ratio between enhanced atom-cavity joint decay and the free space atomic decay rate. It has been shown in this paper that the physical size of the cavity scales linearly with the emission wavelength. An interesting result of this work is that a smaller cavity size as measured in units of $\lambda/2$ results in increased entanglement. The minimum cavity size to obtain the maximum entanglement (joint decay rate) for any atom is $\lambda/2$. In addition to the size of the cavity the cavity enhancement ratio must be on the order of 2×10^5 which corresponds to a cavity loss of 16 ppm. The first is a whispering-gallery-mode (WGM) [19–21] and the second a Fabry–Perot (FP) microcavity [22–25]. Both are available in atom chip technology. The WGM type optical cavity can meet the requirement for high optical reflectivity of around 40 ppm, but may be difficult to place atoms inside since the cavity is made of glass. Additionally, the WGM cavity is multiple wavelengths in circumference ($N \gg 1$) in order to obtain the low cavity losses. The FP type cavity allows placing the atom between the mirrors, but is limited by the relatively high loss of the cavity mirrors of around 200 ppm. Another advantage of using the atom-cavity is that the joint decay rate can be varied by changing the position of one mirror of a FP microcavity using a linear translator. This gives an experimentalist a macroscopic 'knob' to adjust quantum level entanglement. It may be possible to detect the level of entanglement between the atoms in the cavity based on sending a single photon from an entangled pair through the atoms in the cavity [26, 27]. Optical cavities integrated into atom chips hold great promise for experimentally confirming the results of this work and implement scalable quantum computing in the near future.

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