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OPTICAL PHYSICS

# Dynamic magnetic field entanglement stabilization

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**A method for stabilizing oscillations and maximizing entanglement in a decoherence free Heisenberg spin dimer using a time varying magnetic field is presented. Unentangled and fully entangled initial states are investigated. A stabilizing magnetic field intensity function was found for both initial states. These time varying magnetic field intensity functions are different from each other, implying that the magnetic intensity variation needed to stabilize the system depends on the initial state. The time varying magnetic field functions were found using simulated annealing optimization. This work proves that it is possible to remove the oscillatory nature of entanglement and maximize entanglement in decoherence free systems.** 

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## 1. INTRODUCTION

Quantum entanglement is important to implement applications of quantum information and computation. Entanglement is a quantum mechanical phenomenon in which the individual quantum states of a set of particles share the same wave function. Each particle in a set of entangled particles cannot be measured without affecting the other particles. Quantum entanglement has been demonstrated to exist between particles even at very large separations, for instance, distances such as between a satellite and the surface of the Earth. [1].

Heisenberg spin chains are particularly important systems that manifest entanglement phenomena. Physical realizations of spin chains could potentially provide applications to quantum computing [2,3]. Experimentally, many molecules have been shown to behave as spin chains. For instance, $La_2Cu_4$, $Nd_2CuO_4$ [4,5], $CuGeO_3$, $^{13}$C-benzene [6,7], and $LiCuVO_4$ all exhibit behavior similar to the mathematical idealization of the spin chain.

An interesting subject for analysis is the behavior of a Heisenberg spin chain immersed in an external magnetic field. Entanglement in such systems has been studied for various spin chain models [8–11]. In [10], a constant magnetic field of varying magnitudes was able to control the oscillation frequency of entanglement in a Heisenberg spin dimer. As shall be demonstrated in this paper, it is possible to use this effect with a time varying magnetic field to put a decoherence free system into a constant, nearly perfect entangled state. There is active research [12,13] in finding decoherence free systems to implement quantum computing. This paper uses simulation to show that a decoherence free system oscillates between zero and maximum entanglement. The oscillation in entanglement both decreases the percent of time entanglement is available for quantum computation and would be more complicated to implement due to needing to synchronize multiple spin dimers to carry out quantum computations. So, if decoherence is eliminated, the resulting entanglement oscillations of the system will need to be stabilized to efficiently use the entanglement. This paper presents using a time varying magnetic field to stabilize and maximize entanglement to address the anticipated entanglement oscillations in decoherence free systems. To the author's knowledge, this paper is the first to use a time varying magnetic field to stabilize entanglement in a Heisenberg spin dimer system.

## 2. MODEL

In this work, a two qubit XY Heisenberg spin dimer with external magnetic field is considered. The system state is described in terms of a density matrix ($\rho$) that evolves according to Eq. (1):

$$\frac{d\rho}{dt} = -i[H, \rho]. \tag{1}$$

The Hamiltonian $H$ of the spin dimer is given by Eq. (2) [11]:

$$H = B(t)(S_1^z + S_2^z) + J(S_1^+ S_2^- + S_1^- S_2^+) + Jr(S_1^+ S_2^+ + S_1^- S_2^-). \tag{2}$$

This Hamiltonian is described in terms of the parameters: $J$, $r$, and $B$, where $J = (J_x + J_y)/2$, and $r = (J_x - J_y)/(J_x + J_y)$. The terms $J_x$ and $J_y$ denote the exchange interaction between two qubits in each dimension with positive values denoting anti-ferromagnetic interaction and negative values denoting ferromagnetic interaction. The raising and lowering operators for the spin system ($S^{\pm}$) are defined according to $S_n^{\pm} = S_n^x \pm i S_n^y$, $S_n^{(x,y,z)} = \frac{1}{2}\sigma_n^{(x,y,z)}$, with $\sigma_n^{(x,y,z)}$ denoting the

three Pauli spin matrices, and $n$ denoting the $n$th position on the spin chain.

The first term $B(t)S_n^z$ controls the response of the system to the external magnetic field. The external magnetic field is taken to be directed along the $z$ axis, and is a function of time. The second and third terms describe the internal interactions of the system, which are governed by the coupling strength between the spin states ($J$) and the anisotropy of this coupling ($r$).

The entanglement measure evaluated in this work is concurrence [14]. $R$ is constructed from the system density matrix $\rho$ in Eq. (3). The concurrence $C$ is defined by Eq. (4), in which the $\lambda_i$ represent the eigenvalues of the matrix $R$:

$$R = \rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y), \tag{3}$$

$$C = \max\left(\sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}, 0\right). \tag{4}$$

## 3. DYNAMIC MAGNETIC FIELD SIMULATION METHOD

To simulate the effect of a time changing magnetic field on the spin dimer, the elements in the system Hamiltonian that are effected by the magnetic field are changed at each time step according to the magnetic field function of time that is to be simulated. The simulations are carried out using a modified version of the quantum optics toolbox [15] to solve the system of differential equations that determine how the quantum spin dimer system evolves over time. The numerical solver was updated to integrate the set of differential equations described in the Hamiltonian $H$ with a time varying magnetic field, even though in the original form, the numerical solver uses a time-invariant Hamiltonian. The justification for using this approach is that the change of the magnetic field is close enough to infinitesimally small during each simulation time interval, which can be considered as constant over the time interval. Additionally, the elements in the Hamiltonian that are affected by the magnetic field are independent of other components, as shown in Fig. 1. This allows passing all of the time values for the magnetic field as a single vector instead of up to 10 independent vectors. The goals of varying the magnetic field are to reduce the oscillations of the system and maximize the entanglement. The assumed form of the solution is a waveform created from summing 16 different sinusoids. Each of these sinusoids is defined by three parameters: amplitude, phase, and frequency. The optimization method chosen was simulated annealing to find the solution due to the large number of parameters to solve for. The simulated annealing method is relatively robust in avoiding getting trapped in a local maximum. The number of simulations required to find the solution ranged from around 10,000 to 20,000.

For the initial configuration, a constant magnetic field strength of amplitude $a_0 = 0.9850$, joint decay $J = 0.6705$, and ratio $r = 0.4926$ were used. The magnetic field versus time function $B(t)$ consists of two parts. The first part is a time varying magnetic field for 3.2 time units, and the second part is a time independent field of magnitude $a_0$ for the rest of the simulation time. One of the time varying magnetic field functions is plotted in Figs. 2 and 3:

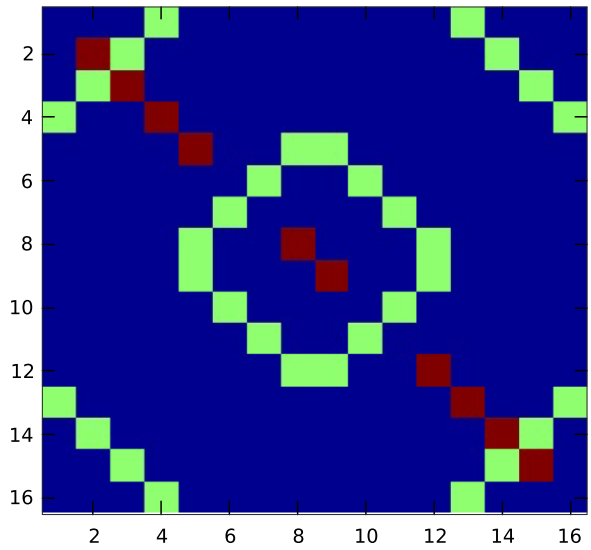


**Fig. 1.** Image of the Hamiltonian matrix, with elements dependent on the magnitude of the magnetic field highlighted in red. The parts of the Hamiltonian not affected by the magnitude of the magnetic field are highlighted in green. The zero magnitude components of the Hamiltonian are highlighted in blue.

$$B(t) = a_o + \sum_{k=1}^{N} a_k \sin(2\pi f_k t + \phi_k). \tag{5}$$

The first method of representing the magnetic field strength consisted of creating the magnetic field strength versus time function by the trigonometric Fourier series with $N = 16$ sinusoidal functions, as shown in Eq. (5). Each sinusoidal function is initially defined by a random amplitude ($a_k$), phase ($\phi_k$), and frequency ($f_k$) which are optimized to yield maximum entanglement. Interestingly, the oscillation frequency of entanglement in this system is proportional to the magnetic field [10]. The practicality of the magnetic field is dependent on the level of technology that can create the highest frequency required in the waveform.

Using the simulated annealing optimization technique with this method yielded 100% for the initially 100% entangled state and 86% entanglement when starting from the unentangled state as measured using the root mean square (RMS). To improve the unentangled state result, a second method to represent $B(t)$ was used. The second method consists of representing the time varying magnetic field using a cubic spline interpolation with 32 control points. This method allowed the optimization to increase the RMS entanglement to 97% for the initially unentangled state. The reason for the better optimization result is that the first method consists of around 49 free parameters that affect the function over the entire 3.2 time units, but the spline technique has only 32 control points that affect only a local area up to the neighboring control points. This reduces the complexity of finding the optimal parameters to create the $B(t)$ function that optimizes entanglement.

The simulated annealing optimization method uses random trials for each initial system state. An example of how the solutions improve over time is shown in Fig. 4 for the initial state of maximum entanglement. Each point in the plot is the RMS of

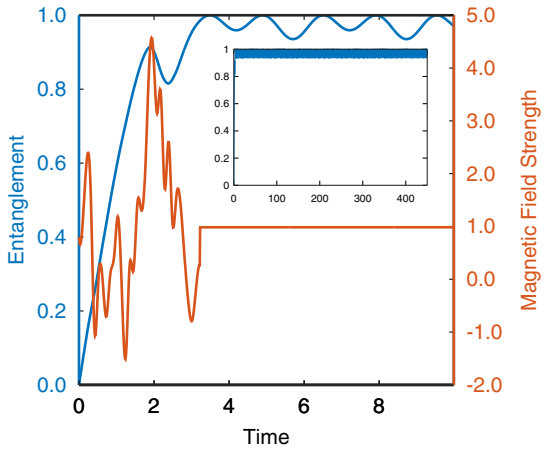


**Fig. 2.** Evolution of entanglement (blue line) with magnetic field as a function of time (orange line) for an initial unentangled state. The inset shows the entanglement over the full simulation time of 450 time units.

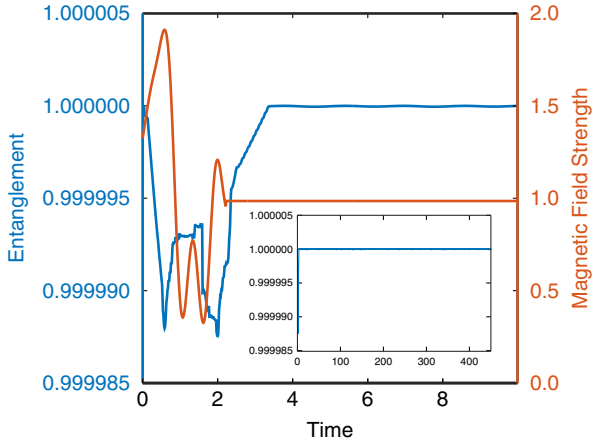


**Fig. 3.** Evolution of entanglement (blue line) with magnetic field as a function of time (orange line) for an initial fully entangled state. The inset shows the entanglement over the full simulation time of 450 time units.

the entanglement over the simulated time and represents the "fitness" of an individual trial solution, as the solver randomly varies the parameters defining the time varying magnetic field strength. The RMS entanglement is used only for optimization purposes in Fig. 4 since it converts the entanglement over the entire simulation time to a single point. The number of parameters varied with each random step increases or decreases depending on the progress of how often the fitness of the solution was improving. For slow improvement, the number of parameters and the standard deviation of the random variation of the parameters are reduced. If no progress is made for a number of trial solutions, the number of random parameters and amount that the parameters are varied were increased by an order of magnitude to break out of a potential local maximum.

## 4. RESULTS

There are a number of results that were interesting from this investigation. The first is that the system entanglement that

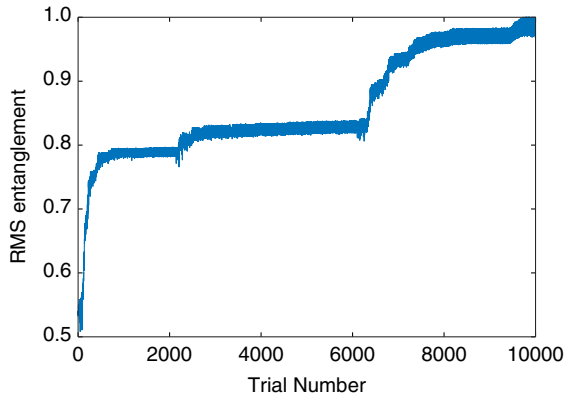


**Fig. 4.** Optimization progress for the RMS value of entanglement with magnetic field as a function of time using cubic spline interpolation and 32 control points for an initially maximum entangled state.

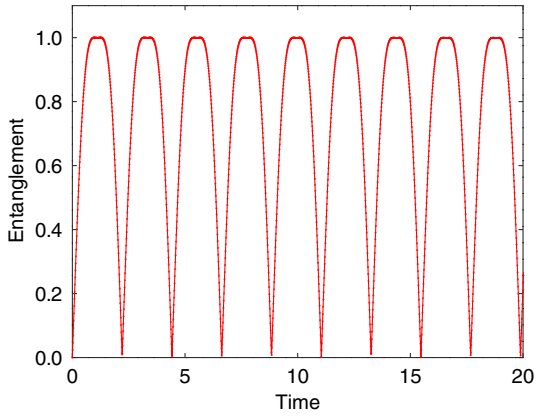


**Fig. 5.** Entanglement oscillation for a decoherence free system using a constant amplitude magnetic field.

normally oscillates between maximum and minimum entanglement, as shown in Fig. 5, can be manipulated with the time varying magnetic field to remove the oscillation and make the entanglement arbitrarily close to full entanglement, as shown in Fig. 3. Second, the optimized magnetic field function of time is dependent on the initial state of the system, as is shown in Figs. 2 and 3. Third, the elements in the Hamiltonian of the system evolution affected by the magnetic field are only on the diagonal and do not overlap the rest of the Hamiltonian, as shown in Fig. 1. This allows updating the Hamiltonian differential equation solver at each time step more efficiently. Finally, the stabilized entanglement created by the initial time varying magnetic field maintains the system in the nearly fully entangled state without oscillations for at least 450 time units. This implies the system will remain in the entangled state for an arbitrary amount of time with no further changes to the magnetic field needed. This will be helpful for maintaining a large number of qubits in a quantum computer.

## 5. DISCUSSION AND CONCLUSION

The time varying magnetic field causes the two spin particles to synchronize and act as if the two particles become one particle in a superposition of the ground and excited states. This implies

there is no way to tell the state of the spin particles until the state is measured. This method of controlling and stabilizing entanglement has the advantage of not requiring any measurement of the quantum spin dimer system. In general, measurement will cause the system to decohere and lose the entanglement that is desired. Other methods such as quantum feedback [9] can both measure the system and restore the coherence, but this requires constant updating of the system. The method of varying the magnetic field also has the advantage that once the initial field is applied, there is no requirement for periodic maintenance of the system coherence. The main drawback, if any, of the time varying magnetic field method is that it requires optimization of many trial solutions for a given initial state of the system. However, the author considers this to be trivial compared with the advantages of no quantum measurement or periodic maintenance required.

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**Data Availability.** Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

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