Feedback enhanced entanglement in a Heisenberg spin chain permeated by a magnetic field

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This paper studies the ability of feedback control to reduce the effect of decoherence and enhance entanglement in an interacting Heisenberg chain model in the presence of an external magnetic field. The system reaches an improved steady state entanglement when feedback is present. The influence of the strength of the external B field is found for optimized steady state entanglement. The time-dependent entanglement evolution of the system is also studied. One potential application of this scheme is to raise the maximum operating temperature of spin entangled systems.

Entanglement is one of the key elements for implementing quantum computation and quantum information. Quantum mechanical entanglement dictates that the state of one entangled particle cannot be measured without affecting the state of the second (twin) entangled particle[1]. Significant research has been carried out with Heisenberg spin systems[2,3], in solid state[4], quantum dot[5] or NMR[6] quantum computation schemes. Additionally, two dimensional atom trapping experiments have been carried out on spin systems for use in quantum computing[7]. A key question for entangled quantum states is how to preserve quantum coherence and entanglement in the presence of decoherence effects. Any quantum system is unavoidably influenced by its environment which gives rises to decoherence processes. Typically the decoherence processes are reduced by carrying out the experiments at single digit Kelvin temperatures. The difficulty in reaching these temperatures is a significant technical challenge[8]. One potential approach to control decoherence to allow operation at a higher temperature is quantum feedback. In this approach, the quantum system of interest is subjected to continuous photodetection, and the information obtained from these measurements is used to achieve a reduction of decoherence in quantum systems.

This letter investigates the improvement in steady state entanglement using homodyne-mediated feedback in a quantum system with interacting Heisenberg spin chain subjected to both the presence of an external magnetic field and decoherence[9]. This work generalizes the analysis of Ref.[3] for this model by treating and controlling decoherence. For any initial state of the system including the state where the two qubits are initially unentangled, the system reaches a improved steady state entanglement when feedback is active. For a given system with fixed spin-spin interaction the level of optimized steady state entanglement is affected by decoherence rate and the external B field. The time-dependent entanglement evolution of the system for some typical initial states is analyzed and contrasted between feedback enabled and disabled. The initial states include an unentangled ground state and an entangled Bell state. The feedback parameters are optimized to achieve the maximum possible concurrence.

I. MODEL DESCRIPTION

The Hamiltonian of a Heisenberg chain of N spin $\frac{1}{2}$ particles with nearest-neighbor interaction is[3]:

$$H_s = \sum_{n=1}^{N} (J_x S_n^x S_{n+1}^x + J_y S_n^y S_{n+1}^y + J_z S_n^z S_{n+1}^z) \qquad (1)$$

where $S_n^\alpha = \frac{1}{2}\sigma_n^\alpha (\alpha = x, y, z)$ is the local spin $\frac{1}{2}$ operator at site n, σ_n^α are the Pauli matrices, and the periodic boundary condition $S_{N+1} = S_1$ applies.

A two qubit system ($N = 2$) will be investigated since it is the simplest spin chain that exhibits entanglement. The Hamiltonian H for an anisotropic two qubit Heisenberg XY system in an external magnetic field B along the z-axis can be written as:

$$\begin{aligned} H_s =& B(S_1^z + S_2^z) + J(S_1^+ S_2^- + S_1^- S_2^+) \\ &+ Jr(S_1^+ S_2^+ + S_1^- S_2^-) \end{aligned} \qquad (2)$$

where $J = (J_x + J_y)/2$, $r = (J_x - J_y)/(J_x + J_y)$, and $S^\pm = S^x \pm i S^y$ are the raising and lowering operators of the spin systems. The parameter $r(-1 \le r \le 1)$ corresponds to the anisotropy of the system and equals 0 for the isotropic XY model and ± 1 for the Ising model. The first term in the Hamiltonian describes the population configuration due to the external magnetic field. The interaction Hamiltonian, which is the second and the third term in Eq.(2) creates coherence between the two qubits which is necessary for the generation of entanglement in this system[3].

The master equation which describes the time evolution of the system in the presence of decoherence is given by

$$\dot{\rho} = -i\left[H_s, \rho\right] + \gamma \mathcal{D}\left[S_1^-\right]\rho + \gamma \mathcal{D}\left[S_2^-\right]\rho \qquad (3)$$

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Here ρ is the density matrix which, in the presence of decoherence, represents the mixed state of the system. The Lindblad super operator $\mathcal{D}$ [10] is defined as $\mathcal{D} = \mathcal{D}[A]B \equiv ABA^{\dagger} - \{A^{\dagger}A, B\}/2$ and describes decoherence from each qubit to the environment. The spontaneous emission rate of the two qubits is represented by γ.

Entanglement between the elements of the Heisenberg spin chain is created by means of the combined influences of anisotropic interactions and a magnetic field B[3]. An initially fully entangled state will reduce to a partially entangled state in the presence of population relaxation. The ability of feedback to counter the effect of population relaxation will be investigated in this letter. The improvement feedback creates is compared in the final steady state entanglement of the system with and without feedback. In order to implement the feedback scheme, a driving laser field is applied to the Heisenberg spin chain. The Hamiltonian of the driving laser field is given by $H_l = \alpha(S_1^y + S_2^y)$, here α is amplitude of the driving field. The fluorescence collected from the system in the y direction will be fed back into the system with a feedback laser field of amplitude $F = \lambda(S_1^y - S_2^y)$.

The master equation rewritten to include homodyne-mediated feedback now becomes:

$$\dot{\rho} = -i\left[H_s + H_l, \rho\right] - \frac{i}{2}\left[\left(S_1^- + S_2^-\right)^{\dagger} F + F(S_1^- + S_2^-), \rho\right] + \gamma\mathcal{D}\left[S_1^+ + S_2^+\right]\rho + \gamma\mathcal{D}\left[S_1^- + S_2^- - iF\right]\rho \tag{4}$$

The steady state solution ρ of master equation given in Eq. 4 allows using concurrence[11–13] as the measure of entanglement. For a system described by the density matrix ρ, the concurrence C is

$$C = \max\left(\sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}, 0\right), \tag{5}$$

where the λ_i are the square roots of the eigenvalues (with λ_1 the largest one) of the "spin-flipped" density operator R, which is defined by

$$R = \rho\left(\sigma_y \otimes \sigma_y\right)\rho^*\left(\sigma_y \otimes \sigma_y\right), \tag{6}$$

where ρ^* denotes the complex conjugate of ρ.

In Eq.4, the amount of feedback can be varied by adjusting the amplitude of F or turned off by setting $F = 0$. The steady state concurrence is maximized by brute force searching for the optimal driving and feedback amplitudes. Once the optimized amplitudes are found these are used to simulate the time evolution of the system using Eq.4.

As shown in Fig. 1, for the two limiting initial states, the corresponding time evolution curves converge to the steady state solution of Eq.(4). Although only two states are presented, the system converges to the same steady state as was simulated for hundreds of initial states. The solid circles on the curves give the time evolution of the concurrence given the steady state optimized feedback. The curves with square markers represent the time evolution of concurrence without feedback. There are a few key points that can be derived from the figure. First, regardless of the initial state of the system, the concurrence reaches a steady state value after some oscillatory behavior. Second, the Heisenberg spin-spin interaction in Eq.(4) creates an entangled state despite the presence of decoherence and absence of feedback. Third, in the case of feedback, the steady state concurrence is reached more quickly as compared with no feedback. Fourth, when feedback is absent, the decoherence limits

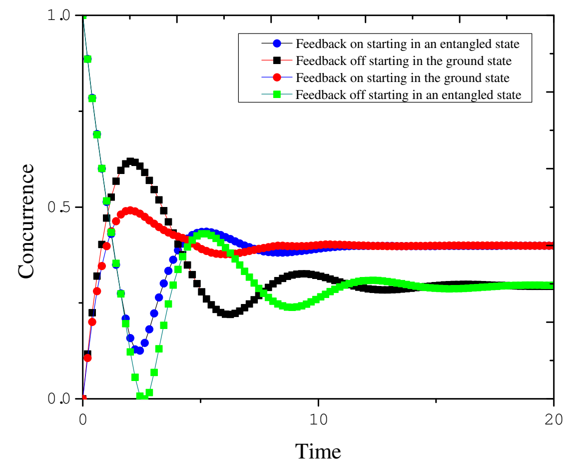

FIG. 1. Plot of the evolution of concurrence versus time for an initially unentangled and a completely entangled state for when feedback is on (circles) and off (squares). For both curves, the system parameters are $\gamma = 0.3$, $J = 0.6$, $B = 0.35$, and $r = 0.5$.

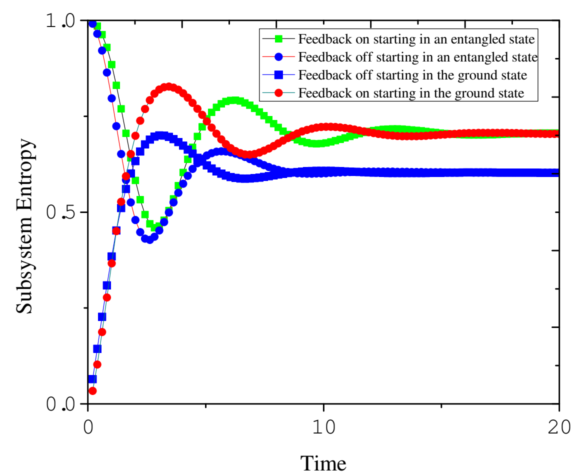

FIG. 2. Plot of the subsystem entropy versus time for an initially unentangled and a completely entangled state for when feedback is on (circles) and off (squares). For both curves the system parameters are $\gamma = 0.3$, $J = 0.6$, $B = 0.35$, and $r = 0.5$.

the steady state concurrence to be approximately 0.3 for either an initially entangled or unentangled state. The steady state concurrence is increased by using feedback to be approximately 0.4. Additionally, as shown in Fig. 2, by using feedback the entropy of the two system eigenstates is decreased from 0.7 to 0.6. The entropy of the subsystem is defined as $S = -tr[\rho_i(log_2\rho_i)]$, here $i = 1, 2$ represents the two eigenstates of the system. This means the purity of the system increases since decreased subsystem entropy implies that one of the system eigenvalues dominates. So, the new physics introduced here is to illustrate that the feedback scheme provides an improvement in both the purity and concurrence of the Heisenberg system. This is in contrast to previous work where an improvement in entanglement did not correspond to an increase in the purity of the system[14].

It is also of interest to show the dependence of the steady state concurrence on the strength of an external magnetic field. As indicated in Fig. 3 and 4, the concurrence increases from $B = 0$ to a maximum when $B = 0.5$ and vanishes as B further increases. The driving and feedback amplitude corresponding to the maximum concurrence are $\alpha = 0.06, \lambda = 0.3$ for the chosen set of parameters $J = 0.6, r = 0.5, \gamma = 0.3$. Since the units used in the derivations are arbitrary, one interesting paper finds the physical J parameter using the spin excitation spectra of a scanning tunneling electron microscope[15]. The experiment was carried out with Magnesium atom chains of length from 1 to 10. The interaction strength J between the atoms was found to be an energy of 6.2 meV. The magnitude found in a paper studying spin interactions in anti-ferromagnetic quantum spin chains was on the order of a 15 kelvin to around 115 Kelvin which is equivalent to 1 to 10 meV[8].

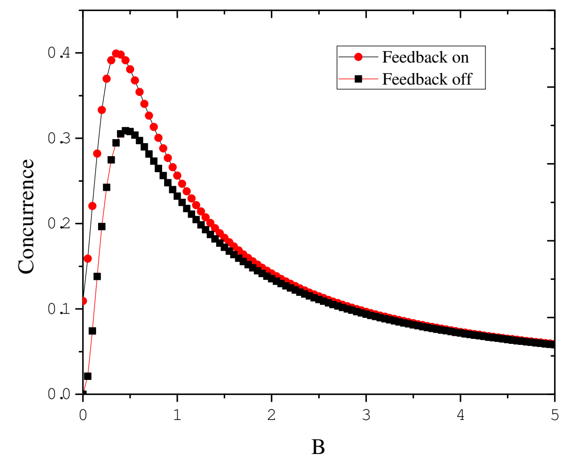

FIG. 3. Plot of the steady state concurrence versus the magnetic field strength $\bar{B}$, where $\gamma = 0.3$, $J = 0.6$, and $r = 0.5$. The line with circle markers represents when feedback is enabled and the line with square markers represents when feedback is off.

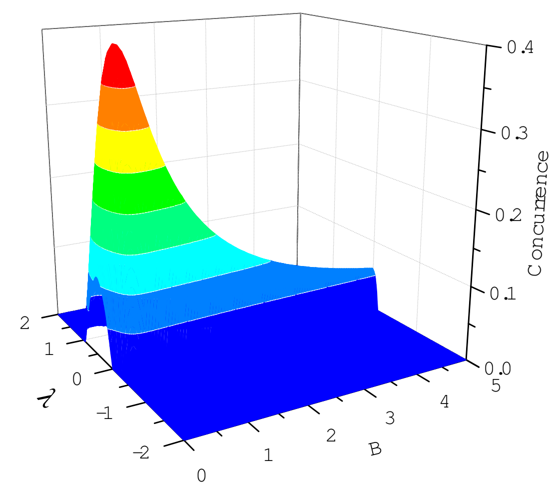

FIG. 4. Plot of the steady state concurrence versus the magnetic field amplitude B and feedback amplitude λ, where $\gamma = 0.3$, $J = 0.6$, and $r = 0.5$.

II. DISCUSSION AND CONCLUSION

Some applications of this theory include explaining the peculiar behavior of the relative magnetic permeability of some materials near absolute zero where the decoherence effects approach those assumed in this paper. A related paper modeled the results of magnetic susceptibility versus magnetic field strength and temperature without feedback using a Heisenberg XY chain with nearest neighbor spin spin entanglement[8]. The experimental model is similar to the model in Eq. 4 except without feedback. Since an increase of temperature tends to decrease entanglement, the temperature effect is equivalent to decoherence. This allows a rough estimate of the temperature equivalence of concurrence in this paper without feedback yields an approximate temperature of 1.35 Kelvin. This is based on comparing the max entanglement of 0.4 in their Fig. 5 showing the quantum correlation versus temperature. The increase of entanglement (0.3 to 0.4) from using feedback is then approximately equivalent to 150 mK. The ability of quantum feedback to create the same amount of entanglement with a higher temperature makes it an interesting path to high temperature entanglement. The open question is "What is the highest temperature, with quantum feedback, where non-negligible concurrence can be achieved?". The answer to this question is a promising path for future quantum entanglement development.

In summary, this paper has given a detailed analysis of using the homodyne-mediated feedback scheme to control decoherence in a Heisenberg spin 1/2 system with an external magnetic field. A steady state and time dependent master equation has been presented to model the driving and feedback amplitude needed to achieve optimal concurrence and purity of the system.

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