On Modeling Microfinance Markets
Mohammad T. Irfan and Luis E. Ortiz
Department of Computer Science, Stony Brook University

PROBLEM STATEMENT

Each microfinance institution (MFI) $i$ has $T_i$ amount of loan to be disbursed. It wants to disburse all of it, at an interest rate $r_i$ determined by the market.

Each $j$-th village wants to maximize the amount of loans obtained.

Where $x_j$ = amount of loan obtained by village $j$ from MFI $i$.

Supply is cleared subject to $0 \leq r_i$.

Non-negative interest rate.

Cannot give more than what it has.

Each microfinance institution (MFI) $i$ has $T_i$ amount of loan to be disbursed. It wants to disburse all of it, at an interest rate $r_i$ determined by the market.

Each $j$-th village wants to maximize the amount of loans obtained.

$\sum_{j \in V_i} x_{j,i} \leq T_i$.

$\sum_{j \in B_i} x_{j,i} \leq g_i(\sum_{j \in B_i} x_{j,i})$.

$\sum_{j \in B_i} x_{j,i}(1 + r_i - e_j) \leq d_j$.

Proof outline: The KKT conditions of the above convex program:

Minimize $\sum_{1 \leq j \leq m} - \log \sum_{i \in B_j} x_{j,i}$, subject to

$\{ \sum_{j \in V_i} x_{j,i} - T_i \leq 0, 1 \leq i \leq n \}$

$\sum_{j \in B_j} x_{j,i} \geq 0, 1 \leq i \leq n$ and $j \in V_i$.

Thus, for the uniform case, an equilibrium point is guaranteed to exist and it is unique!

Primal-dual algorithms exist to solve Eisenberg-Gale convex programs (V. V. Vazirani, Combinatorial algorithms for market equilibria, Algorithmic Game Theory, Chapter 5).

The general case does not necessarily have an equivalent Eisenberg-Gale program.

Counter-example: Consider an MFI operating in two villages.

The revenue generation functions of village 1 and 2 are parameterized by $(d_i, e_i)$ and $(d_j, e_j)$, resp., where $d_i > d_j$ and $e_i < e_j$.

There exist two distinct equilibrium points.

We have derived a sufficient condition for formulating the general case as an Eisenberg-Gale program.

Existence of and algorithms for computation of an equilibrium point in the general case remains open.

SUMMARY OF THEORETICAL RESULTS

The uniform case (all villages having the same revenue-generation function) is equivalent to the following Eisenberg-Gale convex program:

Minimize $\sum_{1 \leq j \leq m} - \log \sum_{i \in B_j} x_{j,i}$, subject to

$\{ \sum_{j \in V_i} x_{j,i} - T_i \leq 0, 1 \leq i \leq n \}$

$\sum_{j \in B_j} x_{j,i} \geq 0, 1 \leq i \leq n$ and $j \in V_i$.

- Each microfinance institution (MFI) $i$ has $T_i$ amount of loan to be disbursed. It wants to disburse all of it, at an interest rate $r_i$ determined by the market.
- Each $j$-th village wants to maximize the amount of loans obtained.
- $x_j$ = amount of loan obtained by village $j$ from MFI $i$.

Thus, for the uniform case, an equilibrium point is guaranteed to exist and it is unique!

Primal-dual algorithms exist to solve Eisenberg-Gale convex programs (V. V. Vazirani, Combinatorial algorithms for market equilibria, Algorithmic Game Theory, Chapter 5).

The general case does not necessarily have an equivalent Eisenberg-Gale program.

Counter-example: Consider an MFI operating in two villages.

The revenue generation functions of village 1 and 2 are parameterized by $(d_i, e_i)$ and $(d_j, e_j)$, resp., where $d_i > d_j$ and $e_i < e_j$.

There exist two distinct equilibrium points.

We have derived a sufficient condition for formulating the general case as an Eisenberg-Gale program.

Existence of and algorithms for computation of an equilibrium point in the general case remains open.

MODEL OF MICROFINANCE MARKET

Each MFI $i$, wants $r_i(T_i - \sum_{j \in V_i} x_{j,i}) = 0$ subject to $0 \leq r_i$.

$\sum_{j \in V_i} x_{j,i} \leq T_i$.

Each village $j$ wants to Maximize $\sum_{i \in B_j} x_{j,i}$ subject to $\sum_{i \in B_j} x_{j,i}(1 + r_i - e_j) \leq d_j$.

Thus, for the uniform case, an equilibrium point is guaranteed to exist and it is unique!

Primal-dual algorithms exist to solve Eisenberg-Gale convex programs (V. V. Vazirani, Combinatorial algorithms for market equilibria, Algorithmic Game Theory, Chapter 5).

The general case does not necessarily have an equivalent Eisenberg-Gale program.

Counter-example: Consider an MFI operating in two villages.

The revenue generation functions of village 1 and 2 are parameterized by $(d_i, e_i)$ and $(d_j, e_j)$, resp., where $d_i > d_j$ and $e_i < e_j$.

There exist two distinct equilibrium points.

We have derived a sufficient condition for formulating the general case as an Eisenberg-Gale program.

Existence of and algorithms for computation of an equilibrium point in the general case remains open.

EMPIRICAL STUDY

Bangladesh microfinance data has been obtained from Palli Karma Sahayak Foundation (PKSF). Below is a snapshot:

We analyze the data of over 450 villages and seven MFIs.

Learn parameters of revenue generation function from data.

Equilibrium computation by soft best-response analysis:

$z_{j,i} = \frac{\exp(-r_i/(1 + r_i - e_j))}{\sum_{j \in B_i} \exp(-r_i/(1 + r_i - e_j))} * (d_i/(1 + r_i - e_j))$.

$T_i = \sum_j z_{j,i} = \frac{\exp(-r_i/(1 + r_i - e_j))}{\sum_{j \in B_i} \exp(-r_i/(1 + r_i - e_j))} * (d_i/(1 + r_i - e_j))$.

Interest rate $r_i$ is computed using binary search.

Section 1: Equilibrium computation based on the original data (representing a very dense underlying network) and on the modified data (representing a sparser network).

Section 2: Intervention analysis by removing government-owned MFIs operating at sub-optimal interest rates and best-response analysis while fixing the government-owned MFIs at their low interest rates.

Section 1: Equilibrium computation based on the original data (representing a very dense underlying network) and on the modified data (representing a sparser network).

Section 2: Intervention analysis by removing government-owned MFIs operating at sub-optimal interest rates and best-response analysis while fixing the government-owned MFIs at their low interest rates.

Future Work

As part of our ongoing work, we are generalizing our model to capture borrowers’ preference for MFIs due to these features as well as interest rates.

Machine learning of this preference is a non-trivial task.

Temporal analysis of Bolivia data with respect to interest rates alone is also of particular interest to us.

Section 2: Intervention

On the other hand, if we fix the actual interest rates of the government-owned MFIs and perform best-response analysis, we find that the interest rates of all other MFIs are driven down (due to competition), but in the end, the government-owned MFIs are subjected to excessive demand.

Computed demand (original demand) for the two government MFIs are respectively 478m (1.3m) and 493m (0.27m). Computed demand matches the original demand for other MFIs.

Analysis of Original Data

Modified Data (Threshold = 25%)

Section 3: Machine learning of this preference is a non-trivial task.

Section 4: As part of our ongoing work, we are generalizing our model to capture borrowers’ preference for MFIs due to these features as well as interest rates.

On the other hand, if we fix the actual interest rates of the government-owned MFIs and perform best-response analysis, we find that the interest rates of all other MFIs are driven down (due to competition), but in the end, the government-owned MFIs are subjected to excessive demand.

Computed demand (original demand) for the two government MFIs are respectively 478m (1.3m) and 493m (0.27m). Computed demand matches the original demand for other MFIs.

Analysis of Original Data

Modified Data (Threshold = 25%)

Future Work

As part of our ongoing work, we are generalizing our model to capture borrowers’ preference for MFIs due to these features as well as interest rates.

Machine learning of this preference is a non-trivial task.

Temporal analysis of Bolivia data with respect to interest rates alone is also of particular interest to us.