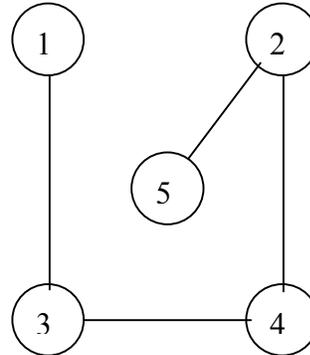


6.034 Fall 2000 Final Problem 3 (Revised¹) Solution and Notes, Constraint Satisfaction

Note -- the Parts of this problem can be done independently.

Consider the following constraint graph. Assume each variable has the same domain $D_i = \{A, B, C\}$. The only valid assignments to pairs of constrained variables are given in the table below.

Constraint (V_i-V_j)	Valid assignments (V_i-V_j)	
1-3	A-C	B-A
2-4	A-A	B-B
3-4	A-B	C-A
2-5	B-A	A-C



Part 1

- A. Do constraint propagation repeated until you achieve arc consistency and show the legal domain values for each variable after the constraint propagation.

Var	Legal Values
1	A B
2	A B
3	A C
4	A B
5	A C

- B. Given only the list of legal values in Part A, what can you say about the number of possible solutions to this problem. Give either a **definite** number or a **definite** range of numbers (specify both the **lower** and upper bounds of the range).

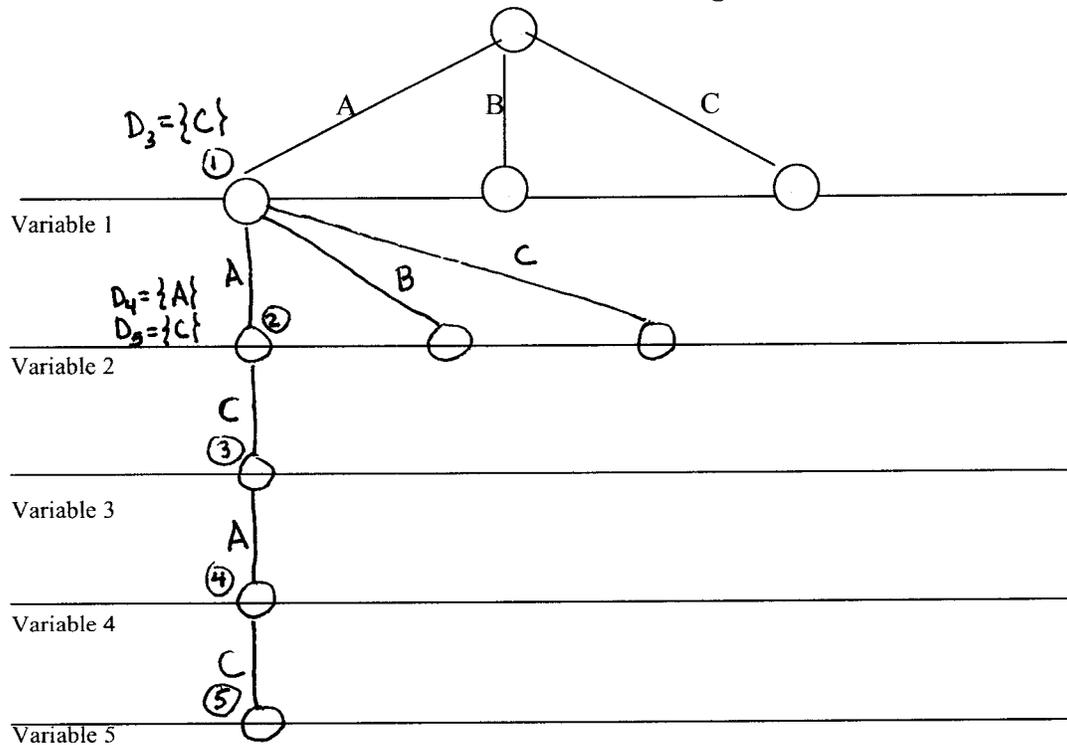
$$0 - 2^5 = 32$$

¹ Part 3 and 4 added (due to Kimberle Koile).

Part 2

In what follows, do not assume that any constraint propagation has been done. Start from scratch!

Find **one** of the valid solutions for this problem using backtracking with forward checking (BT-FC). Examine variables in numerical order and values in alphabetical order. Draw the search tree below; we have started the tree for you. For every node in the tree draw **only** the valid descendants at that point. Each horizontal line indicates the level of the tree corresponding to the assignment of values of the specified variable. **Number the nodes in the tree in the order in which assignments are considered.**

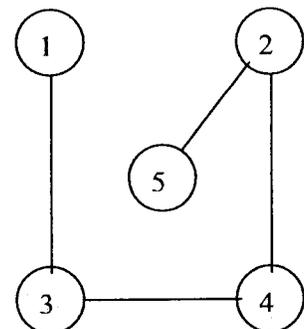


The answer found is:

1	A
2	A
3	C
4	A
5	C

Constraint (V _i -V _j)	Valid assignments (V _i -V _j)	
1-3	A-C	B-A
2-4	A-A	B-B
3-4	A-B	C-A
2-5	B-A	A-C

Repeated for your convenience



Part 2 Explanation

You're supposed to number the nodes in the tree in the order in which assignments are made. This is when the forward constraints for a proposed variable assignment are checked.

Assume that there is no propagation through singleton domains (since that wasn't specified in the problem).

Note: Forward checking is "forward" in the search tree, and the order of variables in the search tree is usually specified (e.g. in numerical order in this problem).

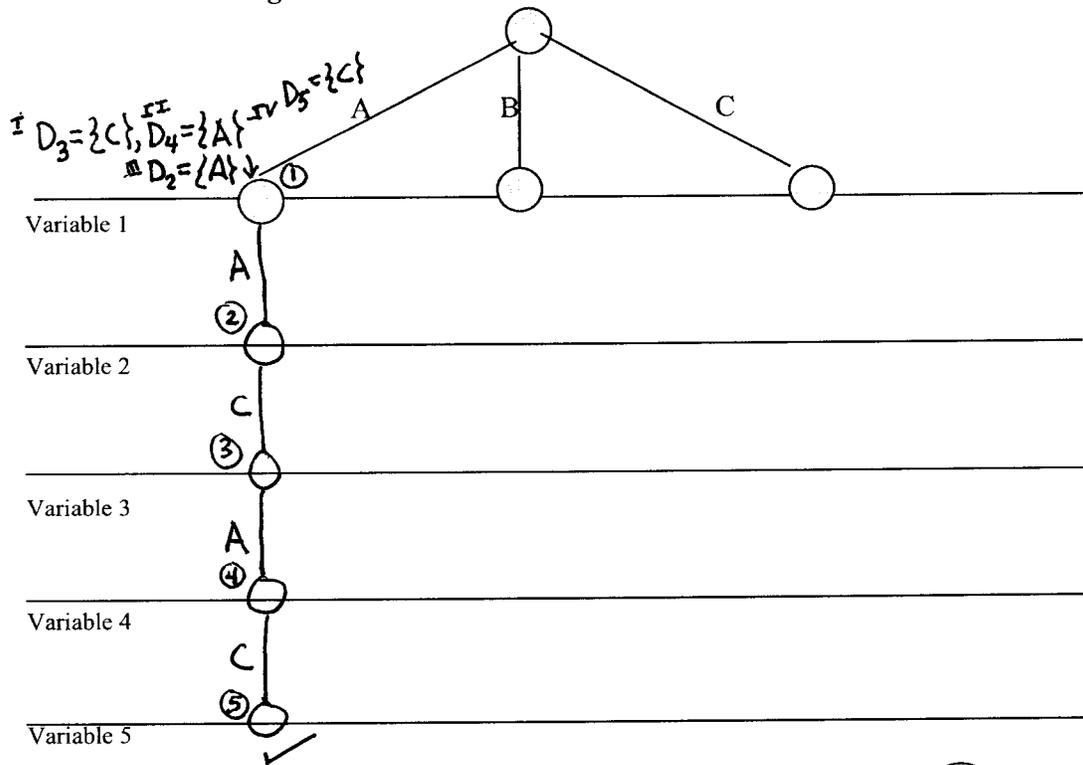
Steps in working this example:

1. Start with var 1: its domain is $\{A B C\}$. Since we're assuming depth-first search, set var 1 = A.
Do forward checking:
 - 1.1 check constraint 1-3; domain of var 3 is reduced to $\{C\}$
ok, so continue
2. Consider var 2: its domain is $\{A B C\}$; with depth-first search, set var 2 = A
Do forward checking:
 - 2.1 check constraint 2-4; domain of var 4 is reduced to $\{A\}$
 - 2.2 check constraint 2-5; domain of var 5 is reduced to $\{C\}$
ok, so continue
3. Consider var 3: its domain has been reduced to $\{C\}$ in step 1.1, so set var 3 = C
Do forward checking:
 - 3.1 check constraint 3-4
ok, so continue
4. Consider var 4: its domain has been reduced to $\{A\}$ in step 2.1, so set var 4 = A
no forward constraints to check (because no constraints between var 4 and var 5)
ok, so continue
5. Consider var 5: its domain has been reduced to $\{C\}$ in step 2.2, so set var 5 = C
no forward constraints to check (because no more variables after var 5)
ok, so done

Part 3 (Revised Part 2.)

In what follows, do not assume that any constraint propagation has been done. Start from scratch!

Find **one** of the valid solutions for this problem using backtracking with forward checking (BT-FC) and propagating through singleton domains. Examine variables in numerical order and values in alphabetical order. Draw the search tree below; we have started the tree for you. For every node in the tree draw **only** the valid descendants at that point. Each horizontal line indicates the level of the tree corresponding to the assignment of values of the specified variable. **Number the nodes in the tree in the order in which assignments are considered.**

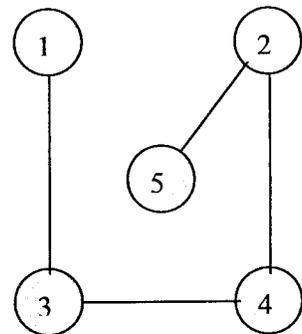


The answer found is:

1	A
2	A
3	C
4	A
5	C

Constraint (V _i -V _j)	Valid assignments (V _i -V _j)	
1-3	A-C	B-A
2-4	A-A	B-B
3-4	A-B	C-A
2-5	B-A	A-C

Repeated for your convenience



Part 3 Explanation

Now consider what happens if we're supposed to propagate through singleton domains. The difference is visible in the answer as the reduced domain of variable 2.

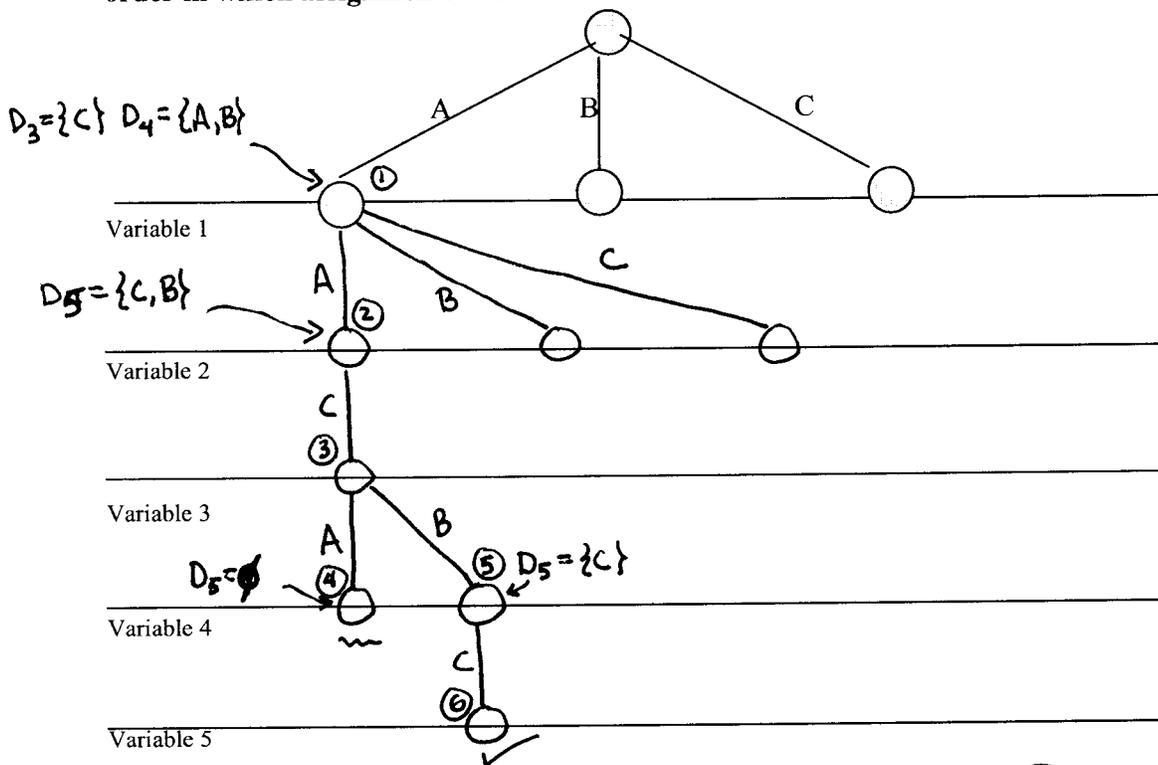
Steps in working this example:

1. Start with var 1: its domain is $\{A B C\}$. Since we're assuming depth-first search, set var 1 = A.
Do forward checking:
 - 1.1 check constraint 1-3; domain of var 3 is reduced to $\{C\}$
 - 1.2 since var 3 domain is a singleton, continue forward checking for var 3:
check constraint 3-4; domain of var 4 is reduced to $\{A\}$
 - 1.3 since var 4 domain is a singleton, continue forward checking for var 4:
check constraint 2-4; domain of var 2 is reduced to $\{A\}$
 - 1.4 since var 2 domain is a singleton, continue forward checking for var 2:
check constraint 2-5; domain of var 5 is reduced to $\{C\}$ok, so continue
2. Consider var 2: its domain has been reduced to $\{A\}$ in step 1.3, set var 2 = A
Do forward checking:
 - 2.1 check constraint 2-4; ok
 - 2.2 check constraint 2-5; okok, so continue
3. Consider var 3: its domain has been reduced to $\{C\}$ in step 1.1, set var 3 = C
Do forward checking:
 - 3.1 check constraint 3-4
ok, so continue
4. Consider var 4: its domain has been reduced to $\{A\}$ in step 1.2, set var 4 = A
no forward constraints to check
ok, so continue
5. Consider var 5: its domain has been reduced to $\{C\}$ in step 1.4, set var 5 = C
no forward constraints to check
ok, so done

Part 4 (Added new valid assignments and constraints to Part 3.)

In what follows, do not assume that any constraint propagation has been done. Start from scratch!

Find **one** of the valid solutions for this problem using backtracking with forward checking (BT-FC) and propagating through singleton domains. Examine variables in numerical order and values in alphabetical order. Draw the search tree below; we have started the tree for you. For every node in the tree draw **only** the valid descendants at that point. Each horizontal line indicates the level of the tree corresponding to the assignment of values of the specified variable. **Number the nodes in the tree in the order in which assignments are considered.**



The answer found is:

1	A
2	A
3	C
4	B
5	C

Constraint (V _i -V _j)	Valid assignments (V _i -V _j)		
1-3	A-C	B-A	
2-4	A-A	B-B	A-B
3-4	A-B	C-A	C-B
2-5	B-A	A-C	A-B
4-5	A-A	B-C	

Repeated for your convenience

