1. Optimal Search

(a) (The first part of this search exercise is due to Kimberle Koile – used here with minor modifications.)

Consider the following search graph.

![Search Graph](image)

The number labeling each link/edge is the cost of the path from the parent/source node of the link to the children/destination node, while the number in parenthesis near a node is the heuristic value for the estimate of the cost of the optimal path from the node to the goal.

i. Compute the optimal path from S to G in the search graph above using the following methods and, for each method, list all the extended partial paths in order. Use lexicographical order to break ties if necessary.
   
   A. British museum
   B. Branch and bound
   C. Branch and bound with an extended list
   D. Branch and bound using the heuristic values
   E. $A^*$

ii. Is the output of $A^*$ optimal? Explain why it is or is not.

iii. If the heuristic values of all the nodes except A are as given in the search graph above, what is the maximum heuristic value for A for which $A^*$ outputs the optimal path regardless of the way ties are broken (i.e., if we use another rule other than lexicographical order to break ties)?

iv. If we change the heuristic value for node B in the search graph above to 7, does $A^*$ outputs the optimal path? Explain your answer.

(b) Consider the following search graph.  

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1This problem was inspired by similar problems by Sourabhi Niyogi.
Express the maximum value for $c$ as a function of $a$ such that A* is guaranteed to output the optimal path to the goal $G$ regardless of the starting node and the tie-breaking rule used to pick partial paths from the queue.

2. Search in Games

Consider the following game tree.

Assume that the children of each node are evaluated from left to right.

(a) Use the Minimax procedure to compute the value of each node in the game tree.

(b) List the nodes that are evaluated using the Alpha-Beta procedure.

(c) Redraw the game tree such that Alpha-Beta performs the minimum number of evaluations (i.e., the best case for Alpha-Beta). How many nodes are evaluated by Alpha-Beta in the resulting game tree?

(d) Redraw the game tree such that Alpha-Beta performs the maximum number of evaluations (i.e., the worst case for Alpha-Beta). How many nodes are evaluated by Alpha-Beta in the resulting game tree?