Recitation 4: Progressive Deepening for Search in Games (Solution)
October 7, 2005

1.

<table>
<thead>
<tr>
<th>Depth</th>
<th>Best Move</th>
<th>Order Alpha-Beta Applied</th>
<th>Num. Evaluations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>static nodes</td>
</tr>
<tr>
<td>1</td>
<td>C</td>
<td>A B C D</td>
<td>3 4</td>
</tr>
<tr>
<td>2</td>
<td>C</td>
<td>A C G H D I B E</td>
<td>4 8</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>A C H G D I N O J K P B E L M F</td>
<td>9 16</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>A B F E L D I N U O C H</td>
<td>5 12</td>
</tr>
</tbody>
</table>

Note how Alpha-Beta did less work at depth 4 than at depth 3, by exploiting information from evaluating up to depth 3 about what the best evaluation order for Alpha-Beta might be (i.e., the node evaluations obtained from Alpha-Beta at depth 3). Using this information does not guarantee an improvement in the performance of Alpha-Beta, but it is very useful as a heuristic.

Below are the trees corresponding to the application of Alpha-Beta at each depth level. The number around a node corresponds to the output of Alpha-Beta after evaluating that node (given also the corresponding values of $\alpha$ and $\beta$ when Alpha-Beta was called for that node). Only those nodes/states where Alpha-Beta was applied (or, in other words, visited by Alpha-Beta) are drawn. The left-to-right order corresponds to the actual order in which Alpha-Beta was applied to the children/successors of the node. Note that, whenever possible, we are using the output evaluations from running Alpha-Beta at the previous depth to order the node evaluations when performing Alpha-Beta up to a particular depth.

For $d = 1$: 

```
MAX

MIN

B 2
C 5
D 4
```

For $d = 2$: 

```
MAX

MIN

MAX

G 5
H 3
I 3
E 1
```
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For \( d = 3 \):

For \( d = 4 \):

Note that, for each depth, the order in which Alpha-Beta was applied to the nodes is exactly the order in which depth-first search would extend the nodes in the corresponding tree above.