Problem 5: SVM (18 Points)

Part A (2 Points)

Consider the two arrangements of + and – points in drawings A and B, in which the arrangement in B is produced by rotating the arrangement in A around the center. Then select true or false for the following assertions:

Given a radial-basis kernel, \( \kappa(v_1, v_2) = e^{-\frac{|v_1-v_2|^2}{0.5}} \), the boundary separating the + and – points in B can be aligned with the boundary in A by a combination of translation and rotation.

- True
- False

Given a polynomial kernel, \( \kappa(v_1, v_2) = (v_1 \cdot v_2)^3 \), where the vectors are drawn from the origin, the boundary separating the + and – points in B can be aligned with the boundary in A by a combination of translation and rotation.

- True
- False
Part B (4 Points)

Assume that a support vector machine is to learn to separate the + and – points in the following diagrams. Sketch the -1 or +1 lines (gutters of the widest street) and circle the points corresponding to support vectors assuming a polynomial kernel, \( \kappa(v_1, v_2) = (v_1 \cdot v_2)^d \)
Part C.1(2 Points)

Assume that a support vector machine is to learn to separate the + and – points in the following diagram. Sketch the decision boundary (the 0 line, not the -1 or +1 lines) and circle the points corresponding to support vectors assuming a radial basis kernel, \( \kappa(v_1, v_2) = e^{-\frac{1}{2\sigma^2}||v_1 - v_2||^2} \), and a very large sigma.

```
+  -
  + + - -
  + -
```

Part C.2(2 Points)

Assume that a support vector machine is to learn to separate the + and – points in the following diagram. Sketch the decision boundary (the 0 line, not the -1 or +1 lines) and circle the points corresponding to support vectors assuming a radial basis kernel, \( \kappa(v_1, v_2) = e^{-\frac{1}{2\sigma^2}||v_1 - v_2||^2} \), and a very small sigma.

```
+  -
  + + - -
  + -
```
Part D (8 Points)

On the separate sheet, there are nine colored diagrams, labeled A through I, representing graphs of SVMs trained to separate pluses (+) from minuses (-). Indicate which diagram results from using which kernel function by writing the letter of the diagram next to the corresponding kernel.

Note that the points are the same in diagrams A, B, C, and D. They are also the same in E, F, and G.

| $\kappa(v_1, v_2) = (v_1 \cdot v_2)^3$ | $\kappa(v_1, v_2) = e^{-\frac{1}{0.5} ||v_1 - v_2||}$ |
|--------------------------------------|--------------------------------------------------|
| $\kappa(v_1, v_2) = (v_1 \cdot v_2)^2$ | $\kappa(v_1, v_2) = e^{-\frac{1}{0.22} ||v_1 - v_2||}$ |
| $\kappa(v_1, v_2) = (v_1 \cdot v_2)^2$ | $\kappa(v_1, v_2) = e^{-\frac{1}{0.08} ||v_1 - v_2||}$ |