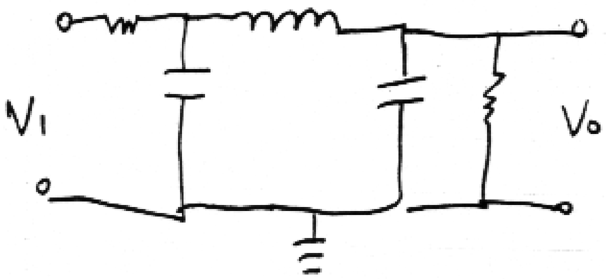
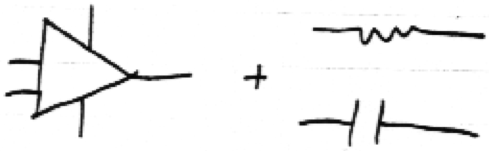
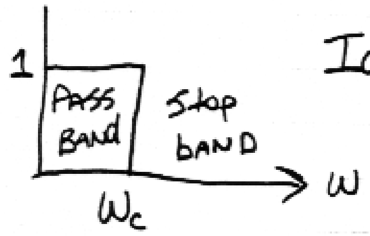
5/11/98



PASSIVE LPF

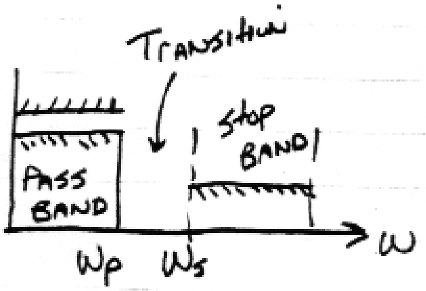


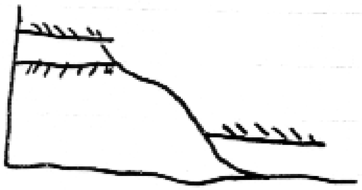
Active RC Filters



Ideal LPF

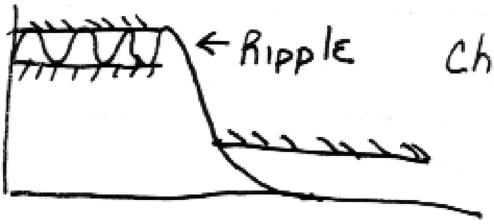
NON-CAUSAL





Butterworth LPF - monotonic

$n_B \equiv$ order of filter

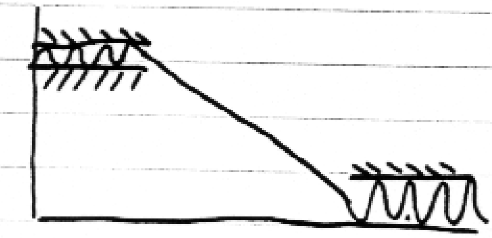


Chebyshev I

$n_{ch}$

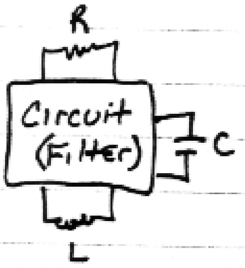
Chebyshev II

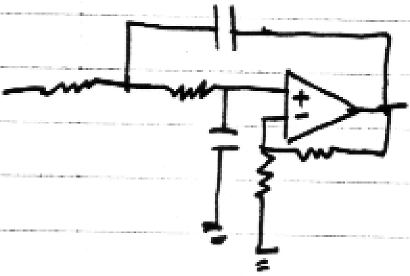
Elliptic



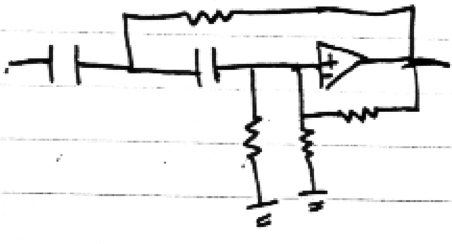
$n_{Elliptic} << N_{Ch}$ ~~&~~ $n_B$

Sensitivity Analysis



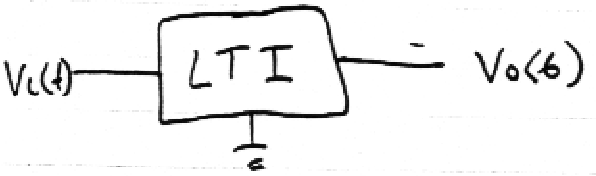


LPF



HPF

# Intro.



$$V_o(t) = T\{V_i(t)\}$$

⇓ Transformation

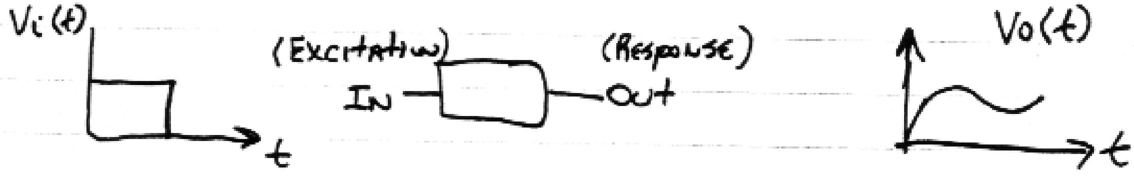
**Linear:** $V_{i_1}(t) \longrightarrow V_{o_1}(t)$

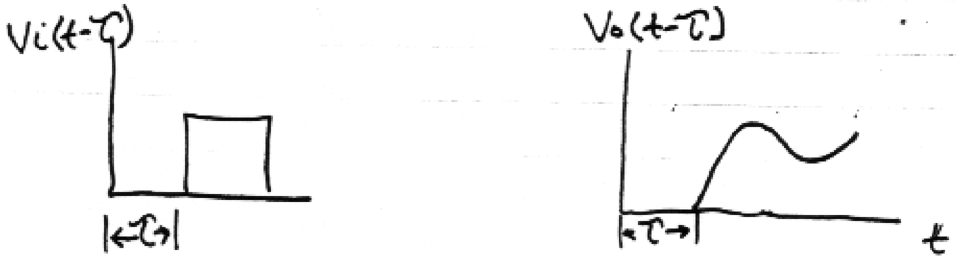
$V_{i_2}(t) \longrightarrow V_{o_2}(t)$

$$\alpha V_{i_1}(t) + \beta V_{i_2}(t) \longrightarrow \alpha V_{o_1}(t) + \beta V_{o_2}(t)$$

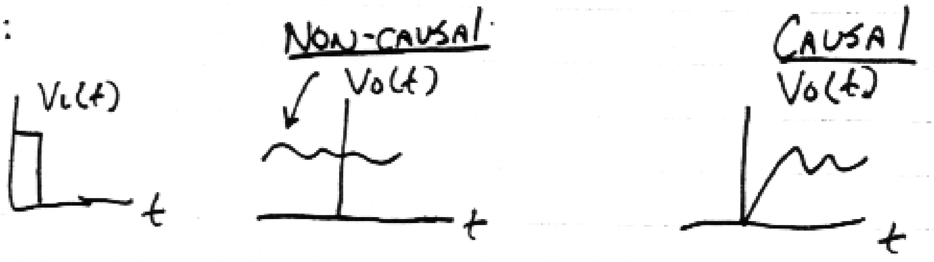
← Superposition →

**Time Invariant**





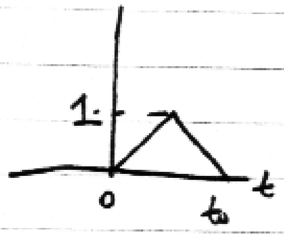
**Causal:**

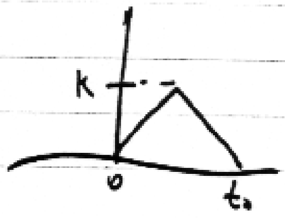


## Examples

Amplifier

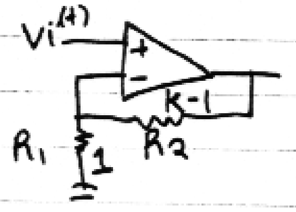
①





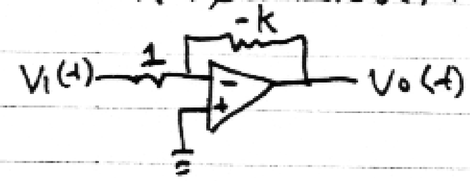
Causal
Linear
Memory-less

→ K > 0 (Non-inverting)



$$\frac{V_o}{V_i} = 1 + \frac{R_2}{R_1} = k$$

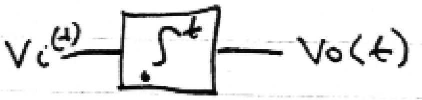
→ K < 0 (Inverting)



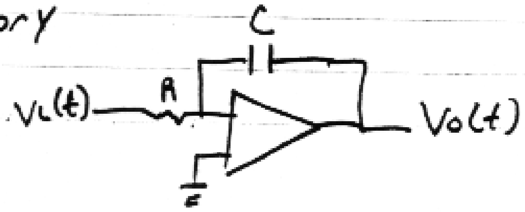
$$\frac{V_o}{V_i} = K$$

→ Integrator

Causal
Linear
Memory



$$V_o(t) = \underset{\text{constant}}{K}\int_0^t V_i(\tau)\,d\tau$$



$$V_o(t) = -\frac{1}{RC}\int_0^t V_i(\tau)\,d\tau$$

# Filter Description

① Time - Domain

② Frequency Domain

$\swarrow$ S - Domain H(s)

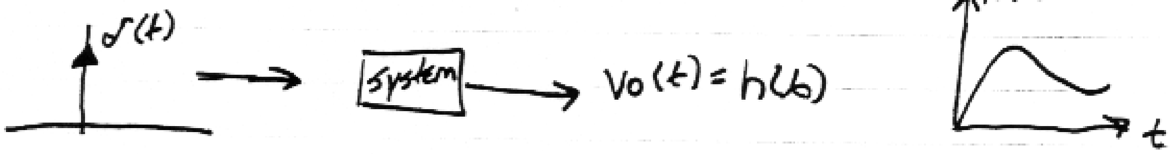
$\downarrow$ Frequency Re

A - Differential Equation

$$\frac{d^2 V_o(t)}{dt^2} + 2 V_o(t) = \frac{d V_i(t)}{dt} - V_i(t)$$

Linear Constant Coeff. Diff. Eq.

B. - Impulse Response



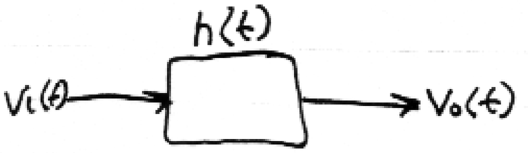
Properties:

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1 \quad \text{(Area)}$$

$$f(t)\,\delta(t-t_0) = f(t_0)\,\delta(t-t_0)$$

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt$$

$$= \int_{-\infty}^{\infty} f(t_0)\,\delta(t-t_0)\,dt = f(t_0)$$



$$= V_i(t) * h(t) = h(t) * V_i(t)$$

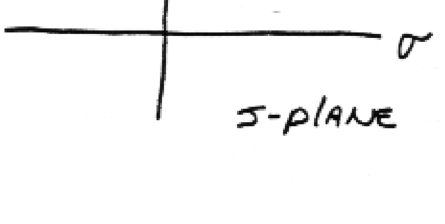
$$= \int_{-\infty}^{\infty} V_i(\tau)\,h(t-\tau)\,d\tau = \int_{-\infty}^{\infty} h(\tau)\,V_i(t-\tau)\,d\tau$$

2 - s-domain representation

$$x(t) \xleftrightarrow{LT} X(s) = \int_{0}^{\infty} x(t)\,e^{-st}\,dt$$

$s = \sigma + j\omega$

$\hookrightarrow$ complex number



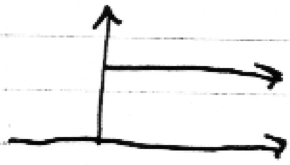
s-plane

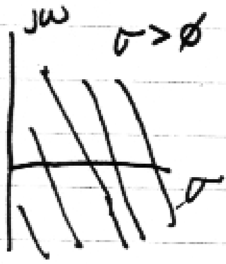
5

R.O.C. - a strip in the s-domain

egion f onvergence

$x(t) = u(t)$





$$X(s) = \int_0^\infty e^{-st}\,dt = \frac{1}{s} \quad \text{R.O.C. } \sigma > 0$$

$$= \int_0^\infty e^{-(\sigma + j\omega)t}\,dt$$

$$= \int_0^\infty e^{-\sigma t} \overset{\nearrow 1}{|e^{-j\omega t}|}\,dt$$

$$|e^{jx}| = \sqrt{\cos^2 x + \sin^2 x} = 1$$

$$\int_0^\infty e^{-\sigma t}\,dt < \infty \quad \sigma > 0$$

---

$$\rightarrow V_i(t) \rightarrow \boxed{\phantom{xx}} \rightarrow V_o(t) = V_i(t) * h(t)$$

LT ↓ LT ↓

$V_o(s) = V_i(s)\,H(s)$ ↗ Transfer function

$$H(s) = \frac{V_o(s)}{V_i(s)}$$

$$\rightarrow \frac{dV_o(t)}{dt} + V_o(t) = V_i(t)$$

$s\,V_o(s) + V_o(s) = V_i(s) \qquad V_o(s)(s+1) = V_i(s) \qquad \dfrac{V_o(s)}{V_i(s)} = \dfrac{1}{s+1}$

$$H(s) = \frac{1}{s+1}$$

6

s-domain representation 5/13/98

$$v_i(t)\ \overset{h(t)}{\boxed{LTI}}\ v_o(t) = v_i(t) * h(t)$$

$$LT \longrightarrow$$

$$V_i(s) \qquad V_o(s) = V_i(s)H(s) \qquad \underset{\Downarrow}{H(s)} = LT\{h(t)\} = \frac{V_o(s)}{V_i(s)}$$

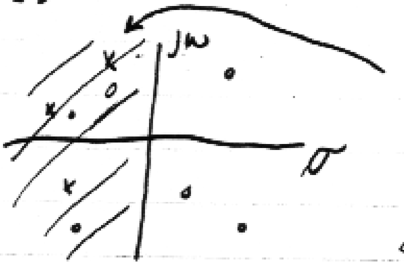
Transfer function

$$H(s) = \frac{N(s)}{D(s)} = \frac{k(s-z_1)(s-z_2)\ldots.(s-z_N)}{(s-p_1)(s-p_2)\ldots.(s-p_N)}$$

$Z_i = \ i = 1, 2, 3, \ldots. m \qquad H(z_i) = 0$ Zeros

$P_\ell = \ \ell = 1, 2, 3, \ldots. N \qquad H(p_\ell) = \infty$ Poles

For a stable filter

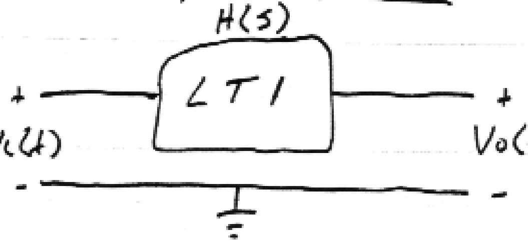


Poles must lie in LHS of s-plane

Zeros may lie anywhere

$$\int_{-\infty}^{\infty} |h(t)dt| < \infty$$

## Frequency Response $H(j\omega)$

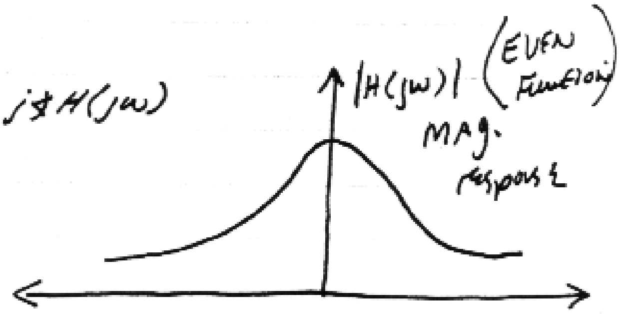


$\alpha\cos(\omega t+\theta) = V_i(t)$

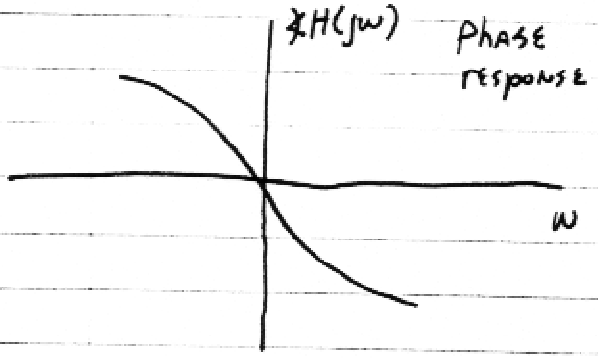
$V_o(t) = \alpha|H(j\omega)|\cos(\omega t+\theta+\measuredangle H(j\omega))$

$$\underset{\uparrow}{H(j\omega)} = H(s)\Big|_{s=j\omega} = |H(j\omega)|e^{j\measuredangle H(j\omega)}$$

Frequency Response

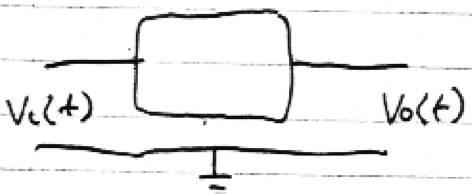


(ODD Function)



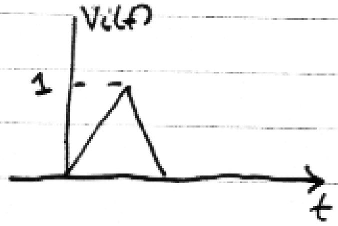
## Types of filters (ideal)

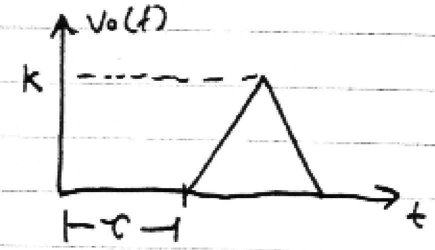
### ① Distortionless Transmission Filter



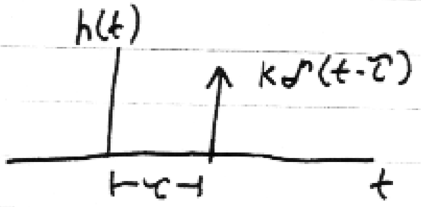
Vo(t) is a delay of Vi(t)

$Vo(t) = K\,Vi(t - \tau)$





① Impulse response, h(t)



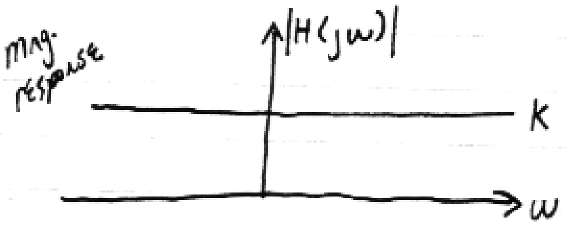
② Transfer function, H(s)

$$H(s) = LT\{h(t)\} = LT\{K\delta(t-\tau)\}$$

$$= Ke^{-s\tau}$$

## ③ FREQUENCY RESPONSE, $H(j\omega)$

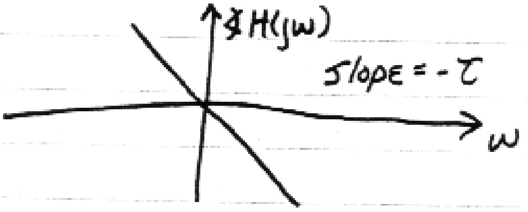
$$H(j\omega) = Ke^{-j\omega\tau}$$



$$|H(j\omega)| = |Ke^{-j\omega\tau}| = K|\overset{1}{\cancel{e^{-j\omega\tau}}}| = K$$

$$\measuredangle H(j\omega) = \measuredangle Ke^{-j\omega\tau}$$
$$= \measuredangle K + \measuredangle e^{-j\omega\tau}$$
$$= 0 - \omega\tau \quad = -\omega\tau$$

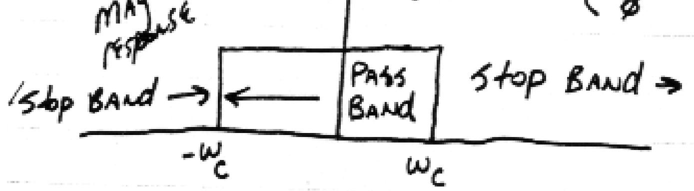
PHASE RESPONSE

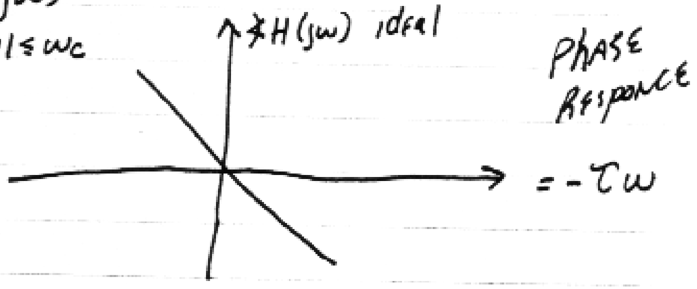


Note: $-$slope $= \tau$ (delay)

## ② Ideal Low Pass Filter, $H(j\omega)$

$$|H(j\omega)|\ \text{ideal} = \begin{cases} 1e^{-j\omega\tau} & 0 \le |\omega| \le \omega_c \\ 0 & \text{O.W.} \end{cases}$$





Ideal $\begin{cases} |H(j\omega)| = 1 \\ \text{for } 0 \le |\omega| \le \omega_c \\ |H(j\omega)| = 0 \ \text{O.W.} \end{cases}$

Test Causality: If $h(t) = 0$ for all $t < 0$, then CAUSAL
O.W. ⇒ NONCAUSAL

Inverse Fourier X-form

$$h(t) = F\overset{\vee}{T}^{-1}\{H(j\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega)e^{j\omega t}\,d\omega$$

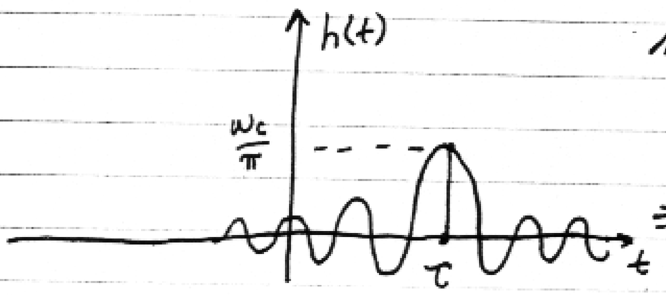
$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\omega\tau}e^{j\omega t}\,d\omega$$

CONTINUE →

$$h(t) = \frac{1}{2\pi}\int_{-w_c}^{w_c} e^{jw(t-\tau)}\,dw \;\boxed{\begin{matrix}\text{X-tra}\\ \text{work}\end{matrix}} \rightarrow h(\tau) = h(t)\Big|_{t=\tau} = \frac{1}{2\pi}\int_{-w_c}^{w_c} dw = \frac{w_c}{\pi}$$

$$h(t)\,(t \neq \tau) = \frac{1}{2\pi}\,\frac{e^{jw(t-\tau)}}{j(t-\tau)}\Bigg|_{-w_c}^{w_c} = \frac{1}{\pi}\left[\frac{\sin w_c(t-\tau)}{(t-\tau)}\right],\ (t \neq \tau)$$

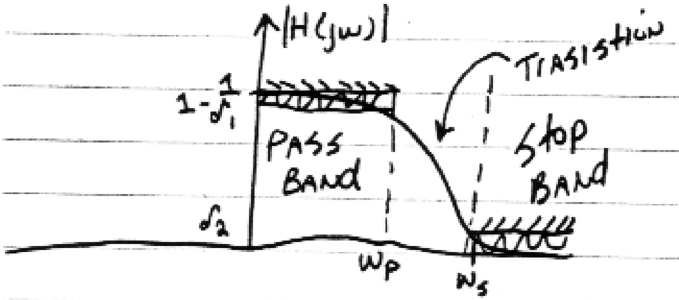
$$h(t) \begin{cases} \frac{1}{\pi}\left[\frac{\sin w_c(t-\tau)}{(t-\tau)}\right], & (t \neq \tau) \\ \frac{w_c}{\pi} & t = \tau \end{cases}$$

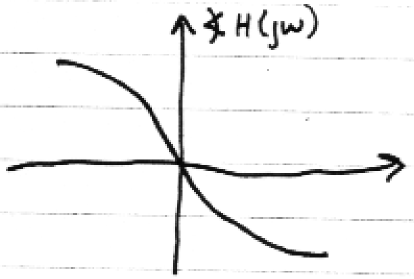


$\Rightarrow$ NON-CAUSAL $h(t) \neq 0$ for all $t < 0$

## Real Filter

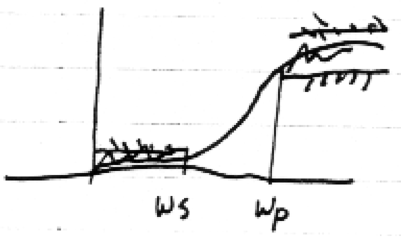
### Low-pass filter



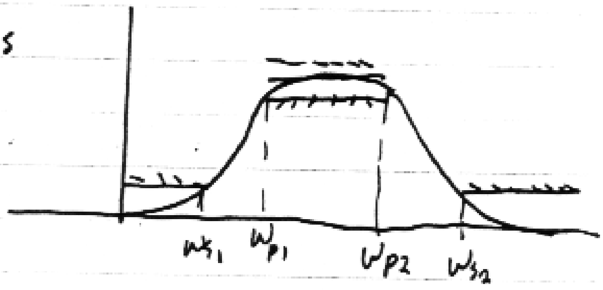


NON-linear

Highpass filter

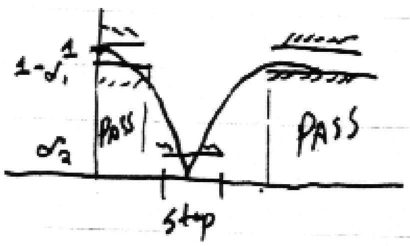


Bandpass

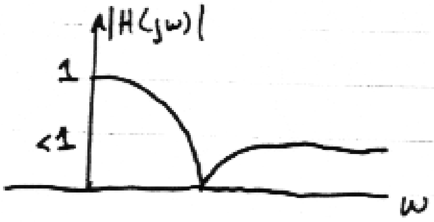


## ④ Band reject - notch

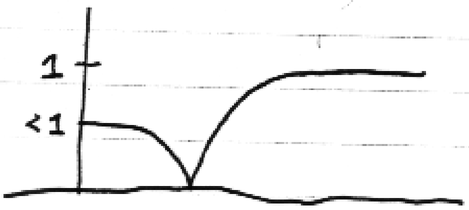
a - Symmetrical Notch



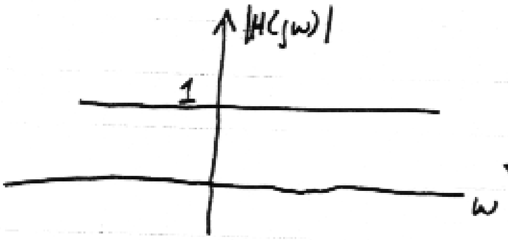
b - low-pass notch

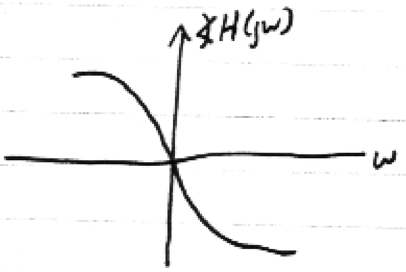


c - High-pass notch



## ⑤ All pass filter





~~Phase~~ PHASE response

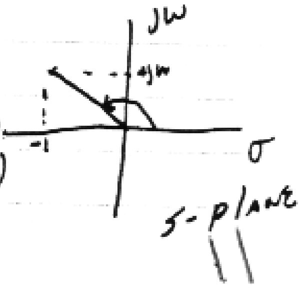
Example:

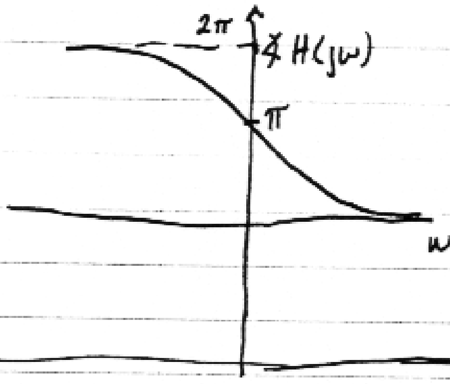
$H(s) = \frac{s-1}{s+1} \Rightarrow$ (1st order all pass)

$H(j\omega) = \frac{j\omega-1}{j\omega+1}$ $\quad |H(j\omega)| = \frac{\sqrt{1^2+\omega^2}}{\sqrt{1^2+\omega^2}} = 1$

$\measuredangle H(j\omega) = \measuredangle(j\omega-1) - \measuredangle(j\omega+1)$

$= \pi - \tan^{-1}(\omega/1) - \tan^{-1}(\omega/1) = \pi - 2\tan^{-1}(\omega/1)$



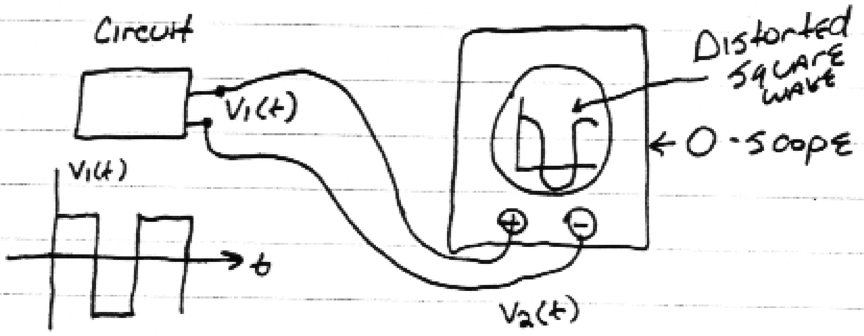


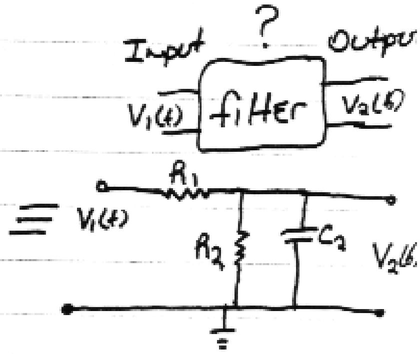
$\tan^{-1}\omega = \pi/2$

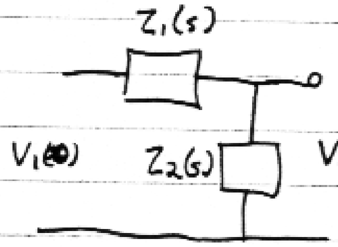
$\omega \to \infty$

$\therefore \measuredangle H(j\omega) \to 0 \quad as\ \omega \to \infty$

## Example:





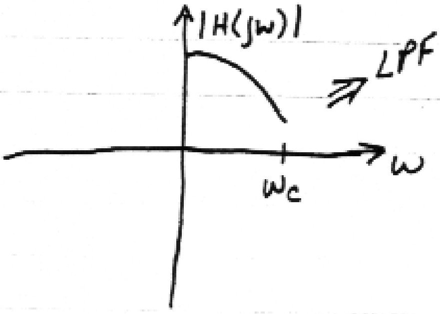


$$H(s) = \frac{V_0(s)}{V_1(s)} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)} = \frac{1}{1+\frac{Z_1(s)}{Z_2(s)}}$$

$$H(s) = \frac{1}{1+Z_1(s)Y_2(s)} = \frac{1}{1+R_1\{\frac{1}{R_2}+sC_2\}}$$

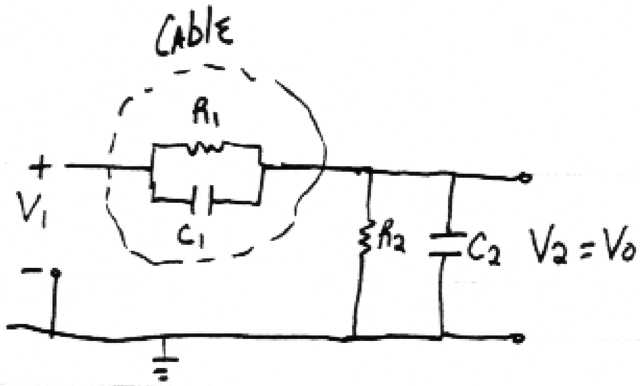
$$H(s) = \frac{1}{1+\frac{R_1}{R_2}+sC_2(R_1)}$$

$$\text{Freq. Resp.} \rightarrow H(j\omega) = \frac{1}{1+\frac{R_1}{R_2}+j\omega C_2(R_1)}$$



What do we do??

- Next Page -



$$H(s) = \frac{1}{1 + \frac{Y_2(s)}{Y_1(s)}}$$

$$H(s) = \frac{1}{1 + \left[\frac{\left(\frac{1}{R_2} + sC_2\right)}{\left(\frac{1}{R_1} + sC_1\right)}\right]}$$

Freq Resp

$$H(j\omega) = \frac{1}{1 + \left[\frac{\left(\frac{1}{R_2} + j\omega C_2\right)}{\left(\frac{1}{R_1} + j\omega C_1\right)}\right]} = \frac{1}{1 + \frac{R_1}{R_2}\frac{(1 + j\omega R_2 C_2)}{(1 + j\omega R_1 C_1)}}$$

↑ make constant

$$\boxed{\text{Choose } R_1C_1 = R_2C_2} \quad \therefore H(j\omega) = \frac{1}{1 + \frac{R_1}{R_2}} = \frac{1}{K}$$

$$\boxed{K = 1 + \frac{R_1}{R_2}}$$

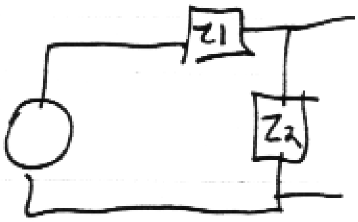
$K = 1, 10, 100, \ldots$

$K = .01, .1, \ldots$

$$\frac{\frac{1}{sC_2}}{R_1+\frac{1}{sC_2}} \quad \frac{1}{sC_2R_1+1}$$

## First Order

$$\frac{V_o(s)}{V_i(s)} = H(s) \qquad \frac{Z_2(s)}{Z_1(s)+Z_2(s)}$$



$$Z_1(s) = R_1 \qquad Z_2(s) = \frac{1}{sC_2}$$

(Pole 1)

$$H(s) = \frac{1}{sC_2+R_1} = \frac{1}{1+sC_2R_1} \qquad P_1 = \frac{-1}{C_2R_1}$$

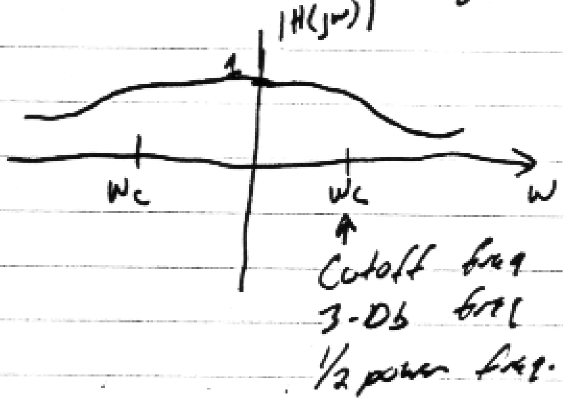
$1+sC_2R_1 = 0$
$s \Rightarrow P_1$
$P_1 = -\frac{1}{C_2R_1}$

## Frequency Response

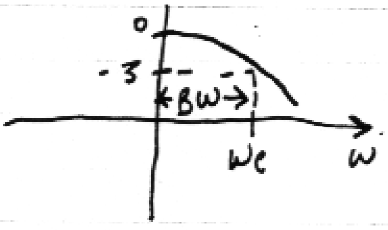
$$H(j\omega) = \frac{1}{1+j\omega C_2R_1} \qquad \tau = C_2R_1 \qquad H(j\omega) = \frac{1}{1+j\omega\tau}$$

## Magnitude Response

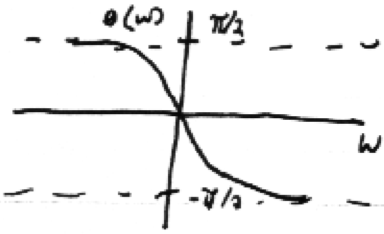
$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega\tau)^2}}$$



$20 \log |H(j\omega)|$



$$\theta(\omega) = \measuredangle H(j\omega) \quad \text{- Phase Response)}$$
$$= \measuredangle 1 - \measuredangle(1+j\omega\tau)$$
$$= -\tan^{-1}(\omega\tau)$$

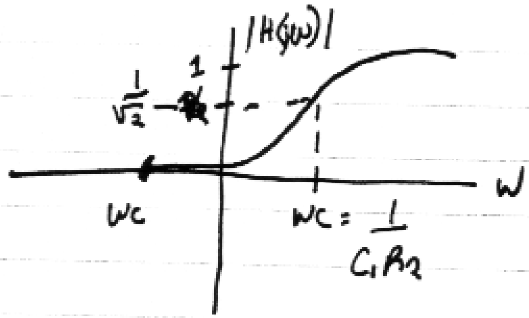


$$\omega_C = \frac{1}{\tau} \Rightarrow \frac{1}{R_1 C_2}$$

$$\theta(\omega_C) = -\tan^{-1}(1) = -\frac{\pi}{4} \qquad -\tan^{-1} \frac{\omega \cancel{\tau}^{1}}{\cancel{\tau}}$$

First Order High PASS

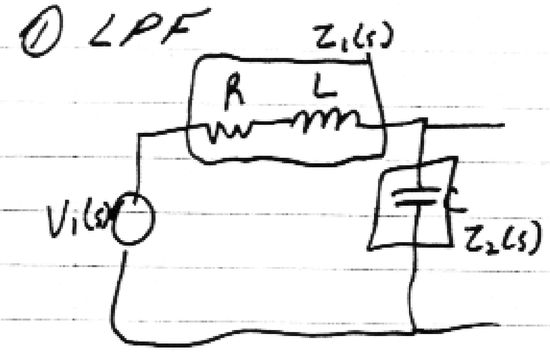
$$H(s) = \frac{R_2}{R_2 + \frac{1}{sC_1}} = \frac{sC_1R_2}{1+sC_1R_2} = \frac{s}{s+\frac{1}{C_1R_2}}$$



$$H(j\omega) = \frac{j\omega}{j\omega + \frac{1}{C_1R_2}}$$

# 2nd Order Filter

① LPF



$$H(s) = \frac{V_0(s)}{V_1(s)} = \frac{Z_2(s)}{Z_1(s)+Z_2(s)} = \frac{1}{1+\frac{Z_1(s)}{Z_2(s)}}$$

$$Z_1(s) = R+sL \quad Z_2(s) = \frac{1}{sC}$$

$$H(s) = \frac{1}{1+sC(R+sL)}$$

$$H(s) = \frac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}} = \frac{\omega_0^2}{s^2+\frac{s\omega_0}{Q}+\omega_0^2} \leftarrow \text{Standard form}$$

$\omega_0^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$ rad/sec (Undamped Natural freq)

$$\frac{\omega_0}{Q} = \frac{R}{L} \Rightarrow Q = \frac{L}{R}(\omega_0) \Rightarrow Q = \frac{L}{R}\left(\frac{1}{\sqrt{LC}}\right) \Rightarrow \underset{\substack{\uparrow \\ \text{Quality Factor}}}{Q} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

## 2nd Order Filter finding Poles

$$s^2+\frac{s\omega_0}{Q}+\omega_0^2 = 0$$

Assume that poles are $s_{p1,2} = -\alpha \pm j\beta$

Two complex conjugate poles

$$s^2+\frac{s\omega_0}{Q}+\omega_0^2$$

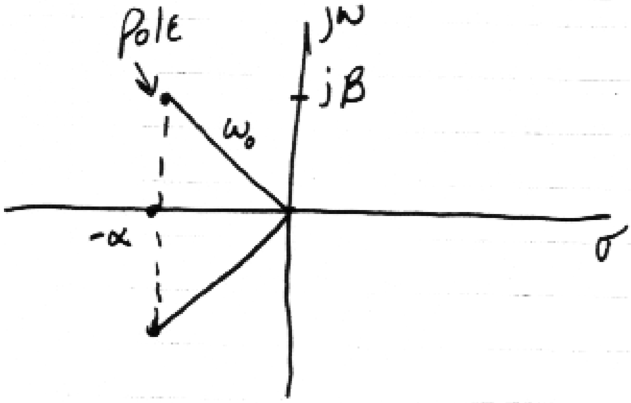
$$(s-s_{p1})(s-s_{p2}) = (s-s_{p1})(s-s_{p1}^*)$$

$\omega_0^2 = S_{p1} S_{p1}^* = \boxed{\alpha^2+\beta^2}$ ①   Note: $(\alpha+j\beta)(\alpha-j\beta) = \alpha^2+\beta^2$

$\frac{\omega_0}{Q} = -\left\{S_{p1} + S_{p1}^*\right\}$

$=$ 2 time real $(S_{p1})$

$\boxed{\frac{\omega_0}{Q} = 2\alpha}$   $\boxed{\alpha = \frac{\omega_0}{2Q}}$   Using 1   $\omega_0 = \sqrt{1-\frac{1}{4Q^2}}$

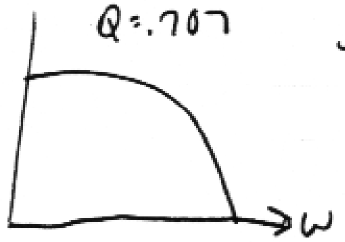


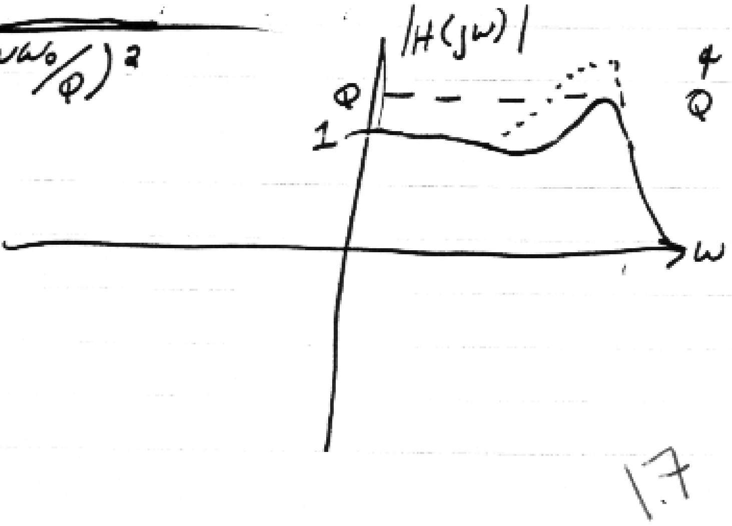
## Frequency Response

$$H(s) = \frac{\omega_0^2}{s^2 + \frac{s\omega_0}{Q} + \omega_0^2} \Rightarrow H(j\omega) = \frac{\omega_0^2}{\underbrace{(j\omega)^2}_{(-\omega^2)} + (j\omega)\frac{\omega_0}{Q} + \omega_0^2}$$

$$|H(j\omega)| = \frac{\omega_0^2}{\sqrt{(\omega_0^2-\omega^2)^2 + (\omega\omega_0/Q)^2}}$$

$H(j\omega_0) =$



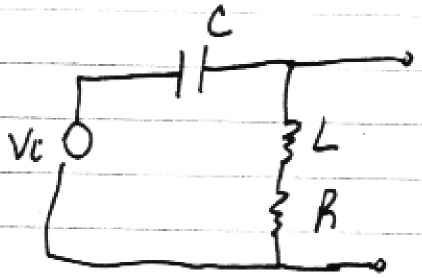


$$H(s) = \frac{H_0(\omega_0^2)}{\sim\sim}$$ → $H_0$ IS A constant

$H_0 = 1$ in past circuit

## HPF (2nd Order)

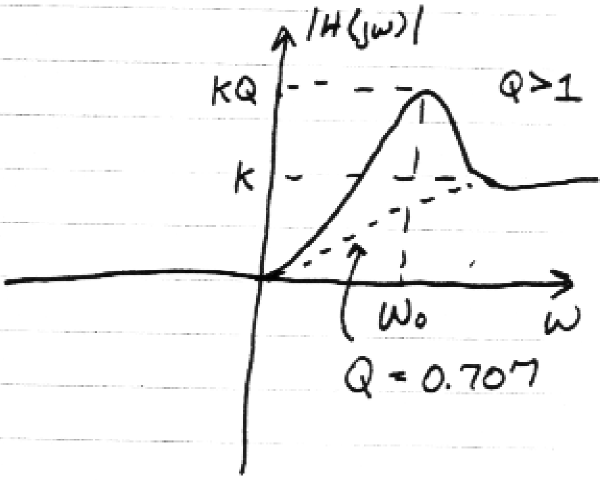
$$H(s) = \frac{Ks^2}{s^2 + \frac{s\omega_0}{Q} + \omega_0^2}$$



## Frequency Response

$$H(j\omega) = \frac{-K\omega^2}{\omega_0^2 - \omega^2 + j\omega\omega_0/Q}$$

$$|H(j\omega)| = \frac{K\omega^2}{\sqrt{(\omega_0^2-\omega^2)+\left(\frac{\omega\omega_0}{Q}\right)^2}}$$

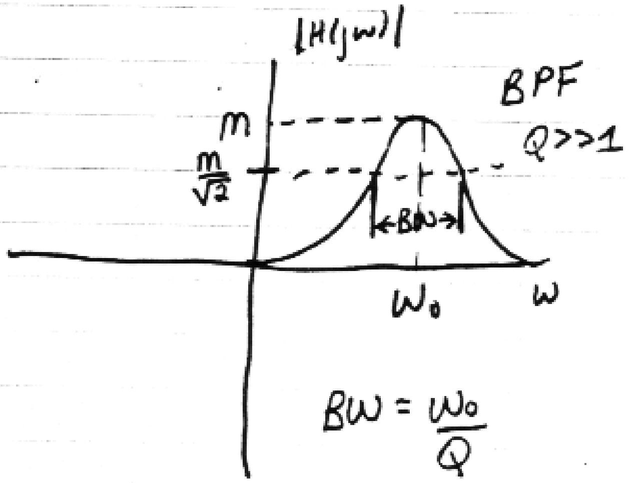


## Bandpass Filter (2nd Order)

$$H(s) = \frac{m\left(\frac{\omega_0}{Q}\right)s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$H(j\omega) = \frac{m\left(\frac{\omega_0}{Q}\right)j\omega}{\omega_0^2 - \omega^2 + j\frac{\omega\omega_0}{Q}}$$

$$|H(j\omega)| = \frac{m\left(\frac{\omega_0}{Q}\right)|\omega|}{\sqrt{(\omega_0^2-\omega^2)^2+\left(\frac{\omega\omega_0}{Q}\right)^2}}$$



$$BW = \frac{\omega_0}{Q}$$

# BAND Stop filter (aka notch)

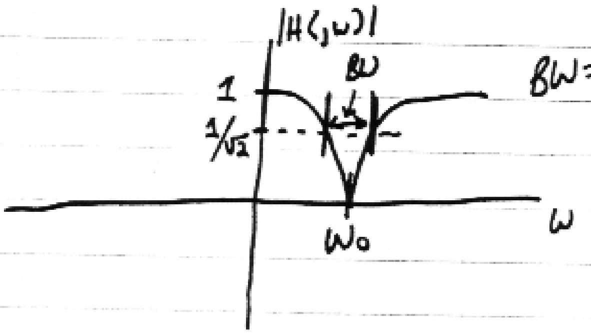
a) Symmetrical

$$H(s) = \frac{s^2+\omega_0^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2} \leftarrow \text{zero's}$$

$$s^2+\omega_0^2=0$$

$$s_{z_{1,2}} = \pm j\omega_0$$

$$|H(j\omega)| = \frac{|\omega_0^2-\omega^2|}{\sqrt{(\omega_0^2-\omega^2)^2+\left(\frac{\omega\omega_0}{Q}\right)^2}}$$



$BW = \frac{\omega_0}{Q}$

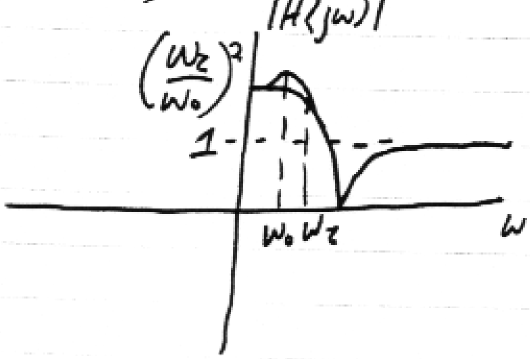
b) Non-symmetrical

LP Notch

$$H(s) = \frac{s^2+\omega_z^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$$

$$|H(j\omega)| = \frac{|\omega_z^2-\omega^2|}{\sqrt{(\omega_0^2-\omega^2)^2+\left(\frac{\omega\omega_0}{Q}\right)^2}}$$

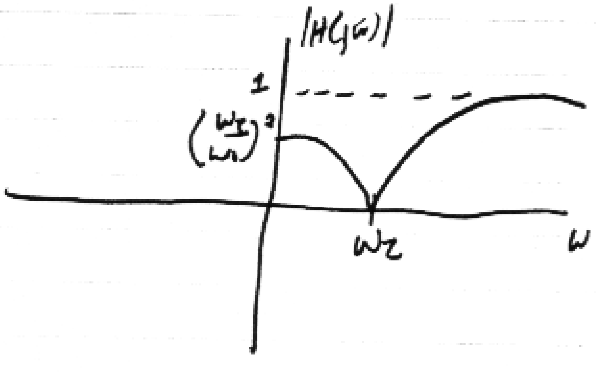
$(\omega_z > \omega_0)$



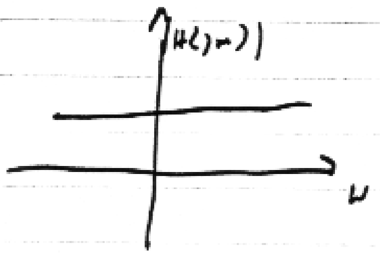
High Pass Notch

$\omega_z < \omega_0$

$$H(s) = \frac{s^2+\omega_z^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$$



# AllPass Filter

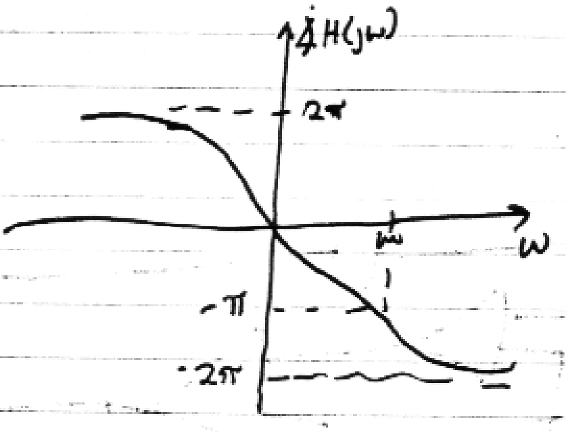


$$H(s)=\frac{s^2-s\frac{\omega_0}{Q}+\omega_0^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$$

$$H(j\omega)=\frac{\omega_0^2-\omega^2-j\omega\omega_0/Q}{\omega_0^2-\omega^2+j\omega\omega_0/Q}$$

$$|H(j\omega)|=1$$

$$\theta(\omega)=\measuredangle H(j\omega)=\measuredangle N(j\omega)-\measuredangle D(j\omega)$$

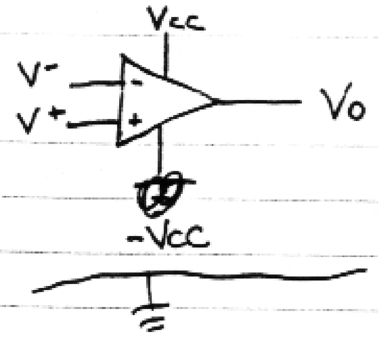


$$=-\tan^{-1}\frac{\frac{\omega\omega_0}{Q}}{\omega_0^2-\omega^2}-\tan^{-1}\frac{\frac{\omega\omega_0}{Q}}{\omega_0^2-\omega}$$

$$\measuredangle H(j\omega)=-2\tan^{-1}\frac{\frac{\omega\omega_0}{Q}}{\omega_0^2-\omega^2}$$

$$\omega=\omega_0 \qquad \theta(\omega_0)=-\pi$$

# The Operational Amplifier



$V_{CC} = 12 - 15V$

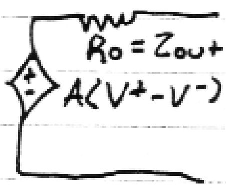
<u>Assume</u> <u>ideal</u> <u>opamp</u>

$$V_o = A(V^+ - V^-)$$

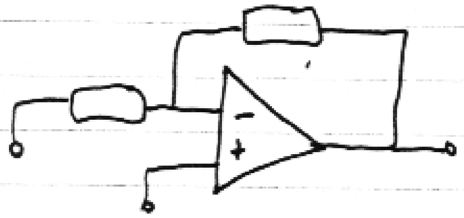
$A \to \infty$ & $A$ is real

$I_N = I_p = 0$   Input impedence $\to \infty$

Output impedence $\to 0$



→ Apply negative feedback in order to render the circuit in the linear region



$$V_o = A(V_p - V_N)$$

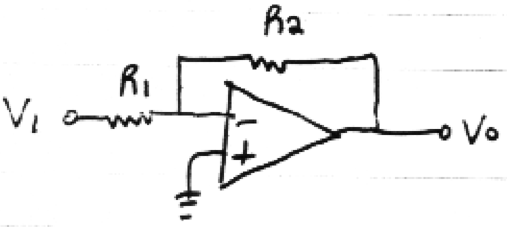
$$\frac{V_o}{A} = V_p - V_N$$

$$\lim_{A \to \infty} \frac{V_o}{A} = 0 = V_p - V_N$$

$$V_p - V_N = 0 \quad V_p = V_N$$

# BASIC Circuits

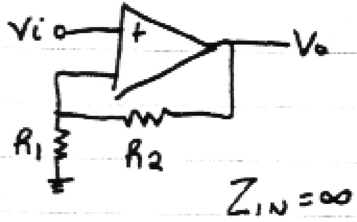
→ Inverting



$$\frac{V_o}{V_i} = \frac{-R_2}{R_1}$$

$Z_{IN} = R_1$

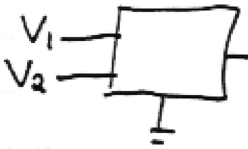
→ Non-Inverting



$$\frac{V_o}{V_i} = 1 + \frac{R_2}{R_1}$$

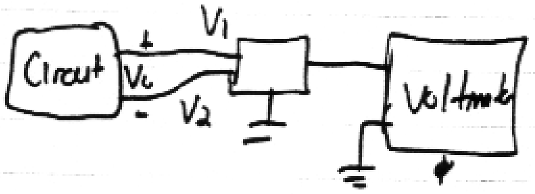
$Z_{IN} = \infty$

→ Differential Amplifier



$V_o = \alpha(V_1) - \beta(V_2)$ where $\alpha$ + $\beta$ positiv

Special case $(\alpha = \beta = 1)$ $(V_o = V_1 - V_2)$

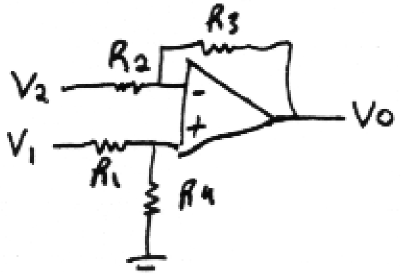


$V_1 = V_i + V_{NOISE}$

$V_2 = V_{NOISE}$ (Common mode noise

$V_o = V_1 - V_2 = V_i + V_{NOISE} - V_{NOISE} = V_i$

– Realization



$V_o = \alpha V_1 - \beta V_2$

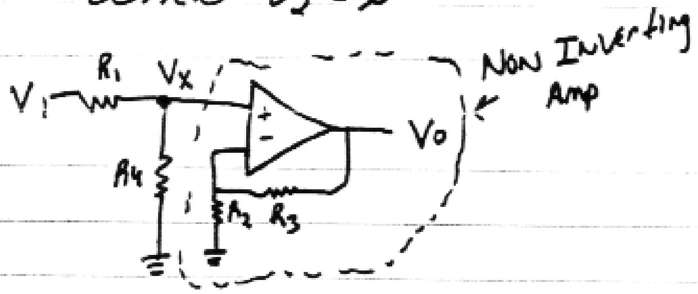
$\alpha = ?$ $\beta = ?$

Linear Circuit

Finding $\alpha$, Assume $V_2 = 0$

$\alpha = \frac{V_o}{V_1}$ if $V_2 = 0$

When $V_2 = 0$



$$\frac{V_o}{V_x} = 1 + \frac{R_3}{R_2} \quad (-1)$$

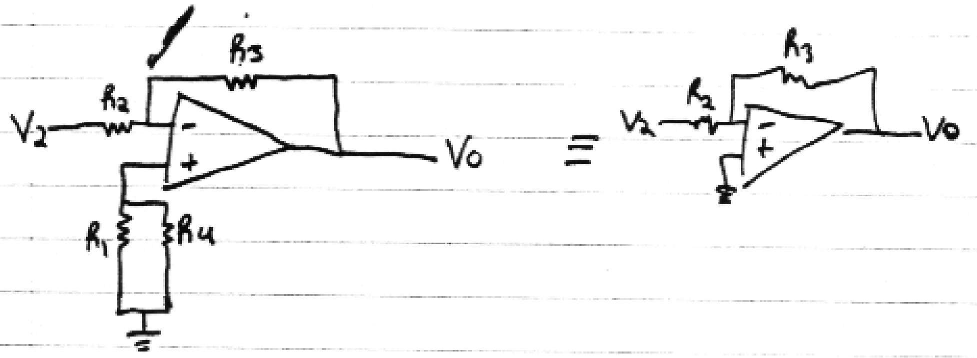
$$\frac{V_x}{V_1} = \frac{R_4}{R_1 + R_4} \quad (-2)$$

$$\alpha = \frac{V_o}{V_1} = \frac{V_o}{V_x}\left(\frac{V_x}{V_1}\right)$$

Divide by $R_4$

$$\alpha = 1 + \frac{R_3}{R_2}\left(\frac{R_4}{R_1 + R_4}\right) = \frac{1 + R_3/R_2}{1 + R_1/R_4}$$

To find $\beta$, $V_1 = 0$ $\quad \beta = \frac{-V_o}{V_2}$



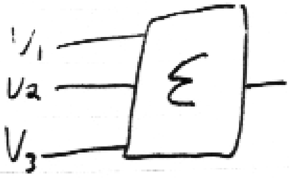
$$\beta = \frac{-V_o}{V_2} = \frac{R_3}{R_2}$$

Special Case $\quad V_o = V_1 - V_2 \Rightarrow \alpha = \beta = 1$

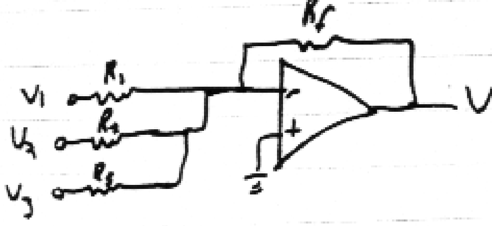
$(\alpha) \quad \frac{R_3}{R_2} = \frac{R_1}{R_4} \qquad (\beta) \quad R_3 = R_2$

$$\therefore R_1 = R_2 = R_3 = R_4$$

## An inverter Adder



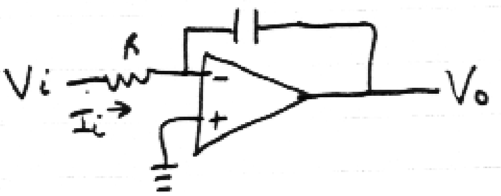
$V_o = -(+\alpha V_1 + \beta V_2 + \gamma V_3)$

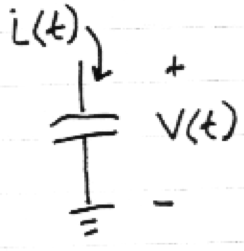


$I_1 = \frac{V_1}{R_1}$ $I_2 = \frac{V_2}{R_2}$ $I_3 = \frac{V_3}{R_3}$

$$\boxed{V_o = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right)}$$

## Integrators





Initial charge

$V(t) = \frac{1}{C}\int_0^t i(\tau)d\tau + V_0$

$I_i = \frac{V_i}{R}$ $V_o(t) = -\frac{1}{C}\int_0^t i(\tau)d\tau + V(0)$

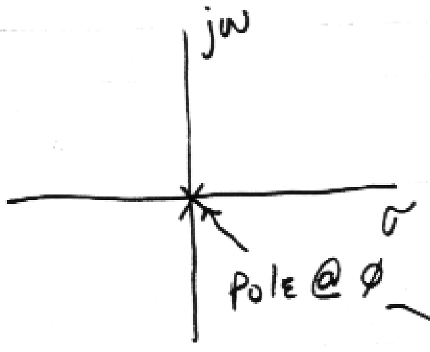
$$\boxed{V_o(t) = -\frac{1}{RC}\int_0^t V_i(\tau)d\tau + V(0)}$$

Assume $V_o(0) = 0$ $V_o(t) = \frac{-1}{RC}\int_0^t V_i(\tau)d\tau$

$\frac{dV_o(t)}{dt} = -\frac{1}{RC}V_i(t)$

$sV_o(s) = -\frac{1}{RC}V_i(s)$

$\frac{V_o(s)}{V_i(s)} = \boxed{-\frac{1}{sCR}}$

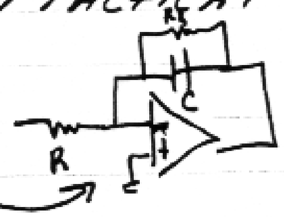


Unstable

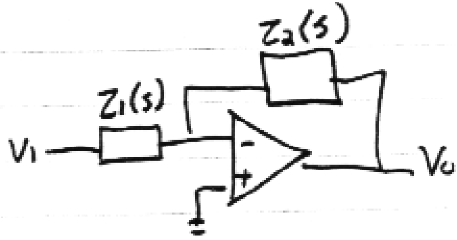
Poles must be on Left H. S.

# Practical Integrator

Lossy Integrator

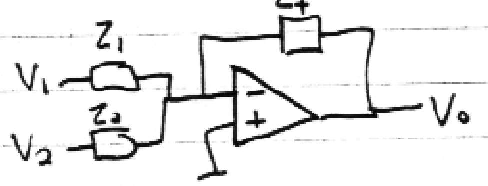


$$R_X >> \left| \frac{1}{\omega_{min} C} \right|$$



$$\frac{V_0(s)}{V_i(s)} = \frac{-Z_2(s)}{Z_1(s)} \qquad Z_1(s) = R \qquad Y_2(s) = \frac{1}{R_X} + sC$$

$$= \frac{-1}{Z_1(s)Y_2(s)} = \frac{-1}{R\left(\frac{1}{R_X} + sC\right)} = \frac{-1}{\left(\frac{R}{R_X} + sCR\right)}$$

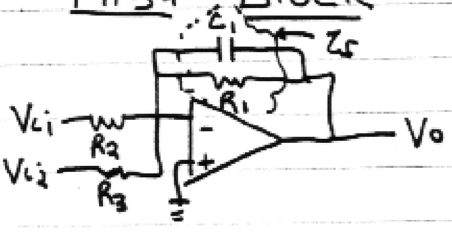


$$V_0 = (\ldots.)V_1(s) + (\ldots.)V_2(s)$$

$$\boxed{V_0 = \frac{-Z_f}{Z_1}V_1 - \frac{Z_f}{Z_2}V_2}$$

## The Biquad Circuit

### First Block

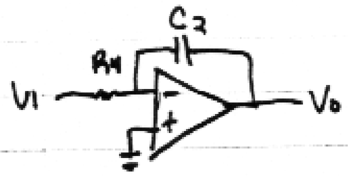


$$V_0(s) = -\frac{Z_f}{R_2}V_{i1} - \frac{Z_f}{R_3}V_{i2}$$

$$= -\frac{1}{Y_f R_2}V_{i1} - \frac{1}{Y_f R_3}V_{i2}$$

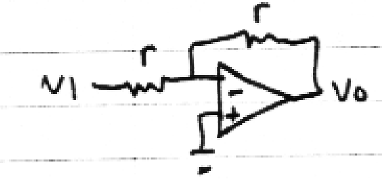
$$= \frac{-1}{\left(\frac{1}{R_1} + sC_1\right)R_2}V_{i1} - \frac{1}{\left(\frac{1}{R_1} + sC_1\right)R_3}V_{i2} \quad ①$$

## 2nd Block



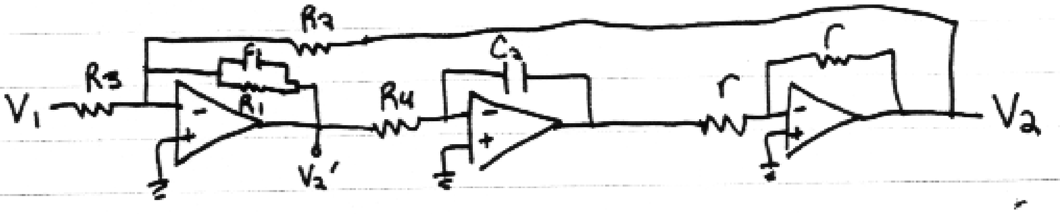
$$\frac{V_o(s)}{V_1(s)} = \frac{-1}{sC_2R_4} \quad (-2)$$

## 3rd Block



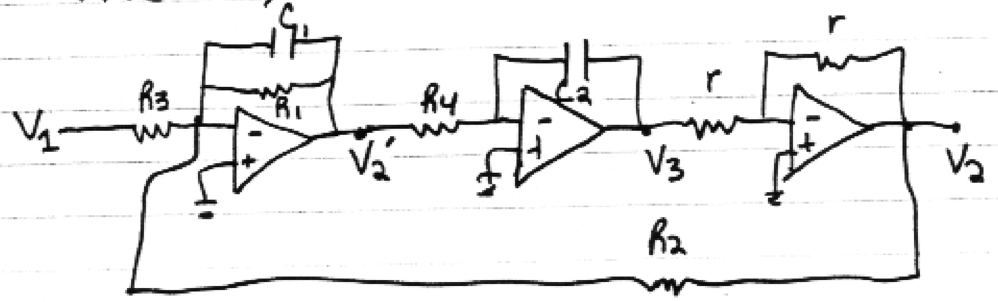
$$\frac{V_o}{V_1} = -1 \quad (-3)$$

## The Complete Circuit



$\frac{V_2(s)}{V_1(s)} = ?$ LRF $\qquad \frac{V_2'(s)}{V_1(s)} = ?$ BPF

# The Biquad Filter



Analysis

$R_2 = R \quad C_1 = C \quad R_1 = QR$

$R_3 = \frac{R}{H} \quad R_4 = R \quad C_2 = C \quad r = R$

$V_2 = -V_3$ ①

$V_3 = \frac{-1}{sC_2R_4} V_2'$ ②

$V_2' = -\left\{\frac{1}{R_3\left(\frac{1}{R_1} + sC_1\right)}\right\} V_1 - \left\{\frac{1}{R_3 + \frac{1}{R_2} + sC_1}\right\} V_2$ ③

$\frac{V_2}{V_1} = ?$

From 1+2

$V_2 = \frac{1}{sC_2R_4} V_2' \Rightarrow V_2' = sC_2R_4V_2$

$sC_2R_4V_2 = -\left\{\frac{1}{R_3\left(\frac{1}{R_1} + sC_1\right)}\right\} V_1 - \left\{\frac{1}{R_3 + \frac{1}{R_2} + sC_1}\right\} V_2$ ④

$$\frac{V_2}{V_1} = \frac{-1/[C_1C_2R_3R_4]}{\underset{\uparrow}{s^2} + s/C_1R_1 + \frac{1}{R_2R_4C_1C_2}}$$

An inverting low-pass filter

$$\frac{-H\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

standard ↑ LPF

$$\omega_0{}^2 = \frac{1}{R_2 R_4 C_1 C_2} \quad \textcircled{-1}$$

$$H = \frac{R_2}{R_3} \quad \textcircled{-2} \quad \frac{\frac{1}{C_1 C_2 R_3 R_4}}{\frac{1}{R_2 R_4 C_1 C_2}} = \frac{R_2}{R_3}$$

$$\frac{\omega_0}{Q} = \frac{1}{C_1 R_1} \Rightarrow Q = C_1 R_1 \omega_0 \Rightarrow Q = C_1 R_1 \frac{1}{\sqrt{R_2 R_4 C_1 C_2}} \quad \textcircled{-3}$$

DESIGN EQUATIONS:

$C_1 = C_2 = C$

$R_2 = R_4 = r = R$

$$\therefore \omega_0 = \frac{1}{RC} \quad \textcircled{-1} \qquad Q = \frac{R_1}{R} \quad \textcircled{3}$$

$$H = \frac{R}{R_3} \quad \textcircled{-2}$$

$$\omega_0 = \frac{1}{RC}$$

$$\frac{V_2}{V_1} = \frac{-H\omega_0{}^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0{}^2}$$ Inverting Low-Pass

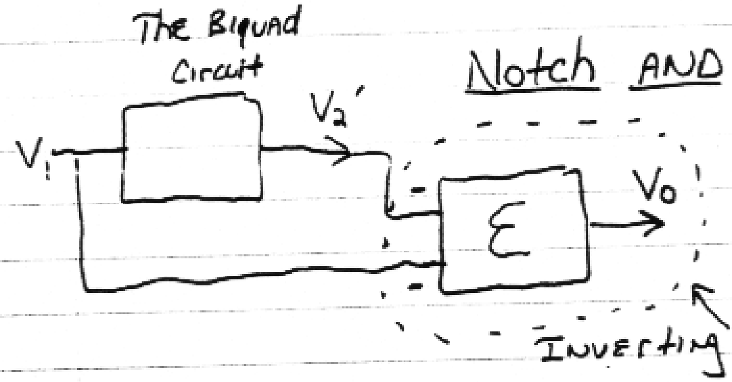
$$\frac{V_3}{V_2{}'} = \frac{-1}{sCR} \qquad \frac{V_3}{V_1} = \frac{-V_2}{V_1} = \frac{H\omega_0{}^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0{}^2}$$ NON-INVERTING LOW PASS

look @ circuit

$$\frac{V_2{}'}{V_1} = \frac{V_2{}'}{V_3} \cdot \frac{V_3}{V_1} = \underset{\substack{\Downarrow \\ \frac{1}{RC} = \omega_0}}{-sCR} \cdot \frac{H\omega_0{}^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0{}^2} = \frac{-H\omega_0 s}{\underset{\uparrow}{s^2 + s\frac{\omega_0}{Q} + \omega_0{}^2}}$$

Inverting BANDPASS Alter

# Notch AND All-PASS Realizations



$$V_o = -(V_2' + V_1) \quad ①$$

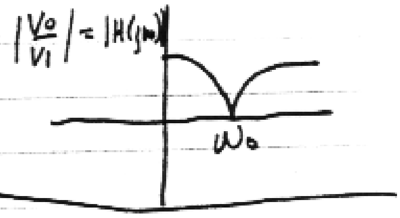
$$\frac{V_o}{V_1} = -1 - \underset{\text{From Before}}{\frac{V_2'}{V_1}} \quad ②$$

$$\frac{V_o}{V_1} = -1 + \frac{\overset{\text{From Before}}{H\omega_o s}}{s^2+s\frac{\omega_o}{Q}+\omega_o^2} = \frac{-(s^2+s\frac{\omega_o}{Q}+\omega_o^2-H\omega_o s)}{s^2+s\frac{\omega_o}{Q}+\omega_o^2}$$

$$= \frac{-(s^2+s(\frac{\omega_o}{Q}-H\omega_o)+\omega_o^2)}{s^2+s\frac{\omega_o}{Q}+\omega_o^2}$$

$$\boxed{\text{Notch Filter} \quad H(s) = \frac{-(s^2+\omega_o^2)}{s^2+s\frac{\omega_o}{Q}+\omega_o^2}}$$

For notch $\frac{\omega_o}{Q} = H\omega_o \Rightarrow H = \frac{1}{Q}$



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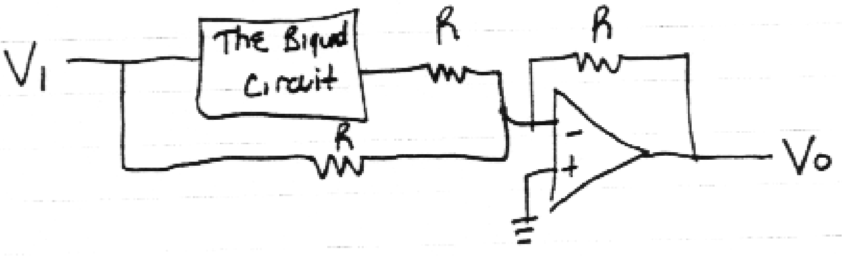
## All PASS

All pass

$$\boxed{H(s) = \frac{-(s^2-s\frac{\omega_o}{Q}+\omega_o^2)}{s^2+s\frac{\omega_o}{Q}+\omega_o^2}}$$

$$H\omega_o - \frac{\omega_o}{Q} = \frac{\omega_o}{Q} \Rightarrow H = \frac{2}{Q}$$



## Example:

Design a BPF with $\omega_0 = 1000$ rad/sec
$BW = 200$ rad/sec
$H = 1$

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$Q = \frac{\omega_0}{BW} = \frac{1000}{200} = 5$ $\omega_0 = \frac{1}{RC} \Rightarrow 1000 = \frac{1}{RC}$

Choose $C = {}_{*}0.1\,uF$
$\therefore R = 10\,K\Omega$

Back to Original Circuit & Fill in values.

$R = 10K$ $C = .1uF$ $QR = 50k\Omega$ $\frac{R}{H} = 10k\Omega$

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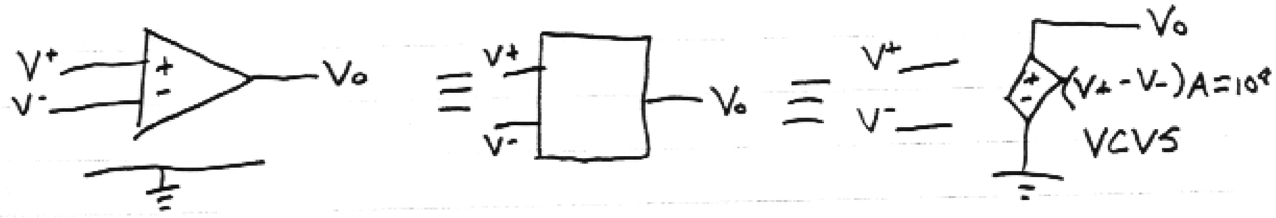


GB → 1 MHz

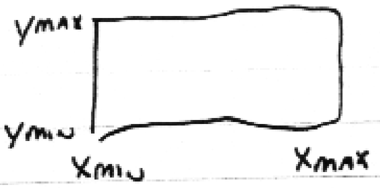
$H(s) = \frac{s^2+100}{s^2+2\boxtimes s+100}$ $\omega_0^2 = 100 \Rightarrow \omega_0 = 10$

$\frac{\omega_0}{Q} = 2\boxtimes \Rightarrow \frac{10}{Q}$ $Q = 5$

```
NUM = [1  0  100];
DEN = [1  2  100];
 w = [0:.1:50];   frequency vector
H = freqs(num, den, w);
mag = abs(H);   mag. response
magdb = 20*log10(mag);   decibels
```

cont →

```
Pha = angle (H);    phase response radians
```



```
V = [Xmin   Xmax   Ymin   Ymax];
AXIS (V);
```

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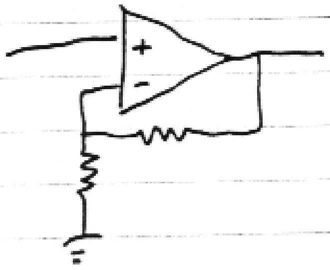
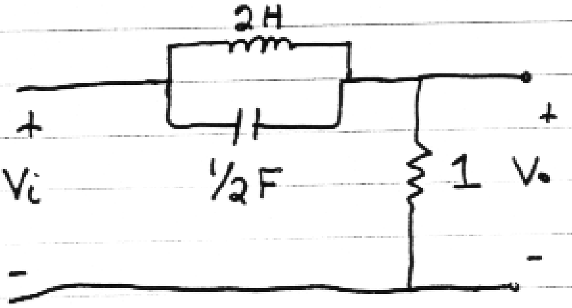
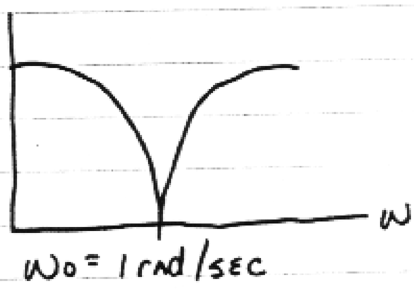


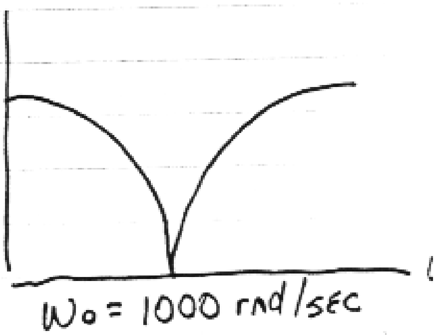
Scaling

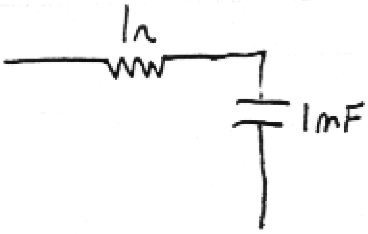
① Frequency scaling (Denormialization)
② Impedence scaling





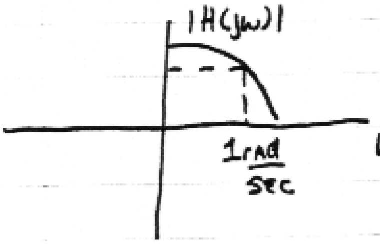
Change L & C to change Wo.



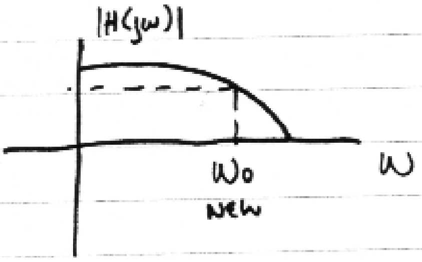


Change to more realistic values.

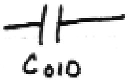
## Frequency Scaling



Modify to



$$H(j\omega) = \frac{f(Z_C(j\omega), Z_L(j\omega), R)}{g(Z_C(j\omega), Z_L(j\omega), R)}$$

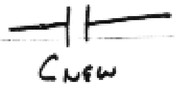


$$Z_{C_{old}} = \frac{1}{j\omega C_{old}}$$

$$Z_{C_{old}}(j1) = \frac{1}{jC_{old}}$$

$$k_f = \frac{\omega_{0(new)}}{\omega_{0(old)}}$$

$$\boxed{C_{new} = \frac{k_f}{C_{old}}}$$



$$Z_{C_{new}}(jk_f) = Z_{C_{old}}(j1)$$

$$\frac{1}{j\omega_{new} C_{new}} = \frac{1}{jC_{old}(1)}$$

$$Z_L = j\omega L$$

$$Z_{L_{old}} = j\omega_{old} L_{old}$$

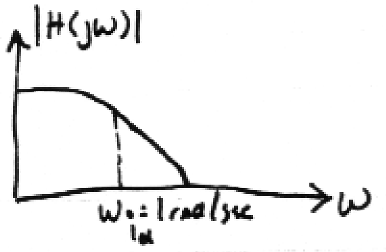
$$Z_{L_{new}} = j\omega_{new} L_{new}$$
$$= jk_f L_{new}$$

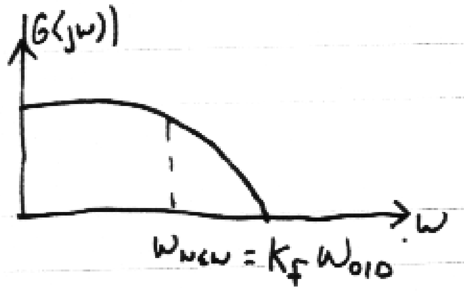
$$\boxed{L_{new} = \frac{L_{old}}{k_f}}$$

$R_{new} = R_{old}$ ← Does not change with frequency

# FREQUENCY SCALING

5/27/98





Lold 

Cold 

Rold 

$$H(jw) = \frac{f\left(jwL_{old}\ \frac{1}{jwC_{old}}\ R_{old}\right)}{G\left(jwL_{old}\ \frac{1}{jwC_{old}}\ R_{old}\right)}$$

$$Z_{old(L)} = jwL_{old}\Big|_{w=1\ rad/sec} = jwL_{new}\Big|_{w=k_f \times 1}$$

$$L_{old} = k_f L_{new} \qquad \boxed{L_{new} = \frac{L_{old}}{k_f}}$$

Kf - frequency scaling factor (Denormalization factor)

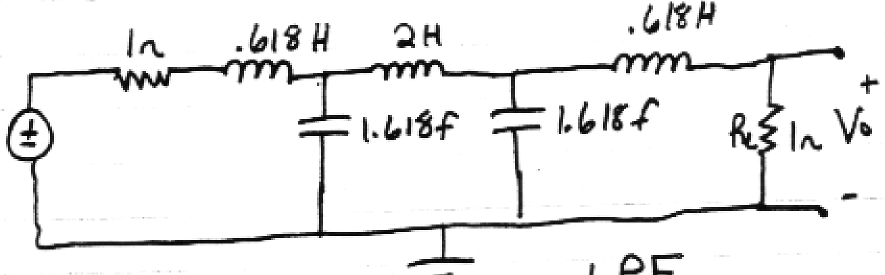
$$Z_{old(C)} = \frac{1}{jwC_{old}}\Big|_{w=1} = \frac{1}{jwC_{new}}\Big|_{w=k_f \times 1}$$

$$\boxed{C_{new} = \frac{C_{old}}{k_f}}$$

$$\boxed{R_{new} = R_{old}}$$

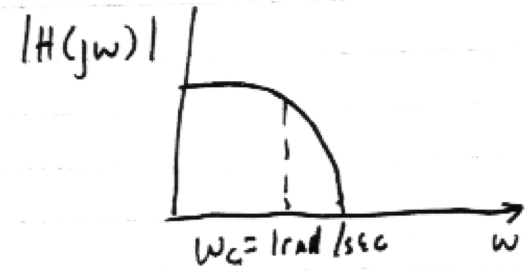
## Example

$W_C = 1 \text{ rad/sec}$



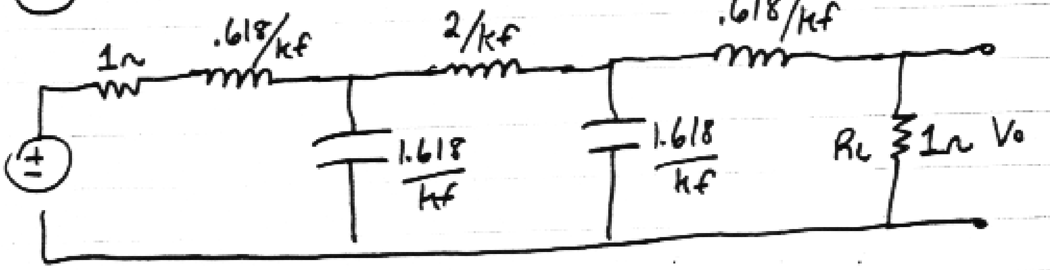
LPF

Denormalize the circuit such that $W_C = 10^4$ rad/sec



$$K_f = \frac{W_{c(new)}}{W_{c(old)}} = \frac{10^4}{1} = \boxed{10^4}$$

$$L_{new} = \frac{L_{old}}{k_f} \quad \frac{.618}{10^4} = \qquad C_{new} = \frac{C_{old}}{k_f}$$



Now we need to scale all values for practical ones

$Z_{NEW} = k_m Z_{OLD}$ at same freq.

If $R_L$ must $= 1k\Omega$

then $K_m = 1000$

$R_N \rightarrow k_m R_o$

$j\omega L_{NEW} \rightarrow k_m j\omega L_{OLD} \quad \therefore L_N \rightarrow k_m L_{OLD}$

$\frac{1}{j\omega C_{NEW}} \rightarrow k_m\left(\frac{1}{j\omega C_{OLD}}\right) \therefore C_N \rightarrow \frac{C_{OLD}}{k_m}$ ⟸ ∴ scale all component values

## Prototype Circuit

$W_c = 1$ rad/sec

$K_f = \frac{W_{c(new)}}{W_{c(old)}}$

$R_{new} = K_m R_{old}$

$K_m \equiv$ impedance scaling factor

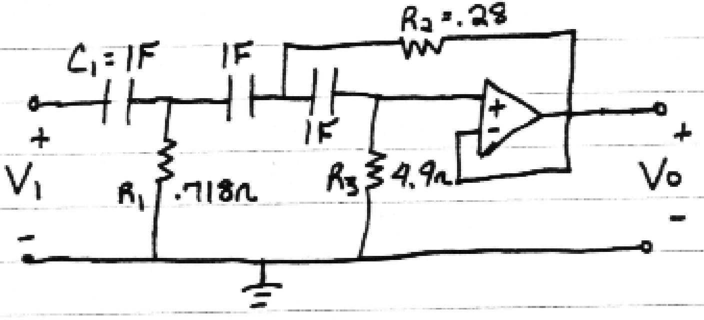
$L_{new} = \frac{L_{old}}{K_f K_m}$

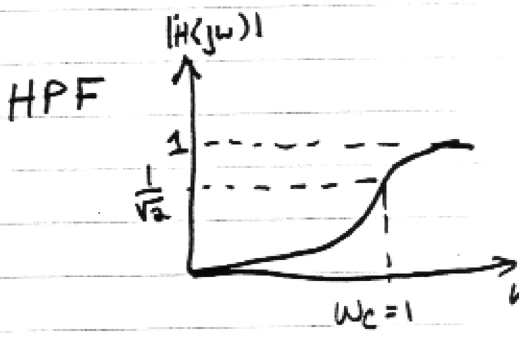
$C_{new} = \frac{C_{old}}{K_f K_m}$

$Amp_{old} = Amp_{new}$

Example:

Design a third order HPF such that the 3db cutoff freq. is 200 Hz. given that only 0.1uF caps are available. Choose an active realization.

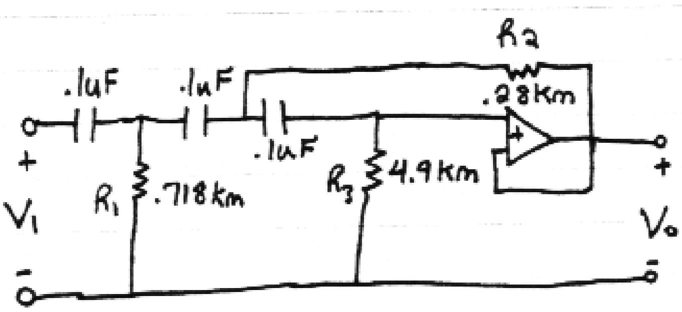




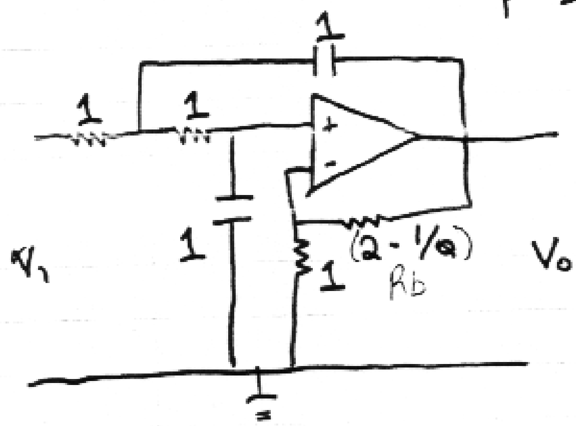
$k_f = \frac{2\pi(200)}{1} = \boxed{4\pi \times 10^2}$

$K_m \Rightarrow C_{old} \rightarrow C_{new}$ $\quad C_{new} = \frac{C_{old}}{k_f K_m}$

$K_m = \frac{1}{k_f \times .1 \times 10^{-6}} = \boxed{795}$



# PSPICE iT



⇓ $W_c = 1$ rad/sec → $W_{c_{NEW}} = 2\pi \times 10^3$

with c = 10nF

with Q = 0.707

$C_N = \frac{C_o}{K_f K_m}$

$\therefore \boxed{K_f = 2\pi \times 10^3}$

$K_m = 15915.5$

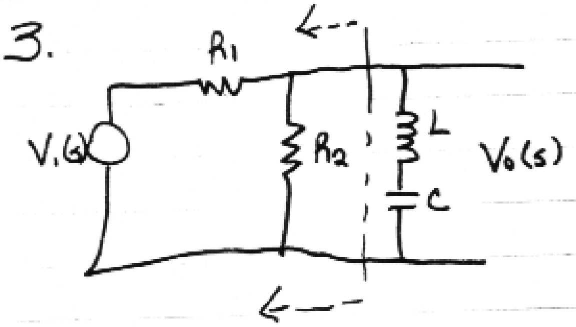
$K_m = \frac{C_{old}/C_{new}}{K_f}$

$.000000010 = \frac{1}{2\pi \times 10^3 \; km}$

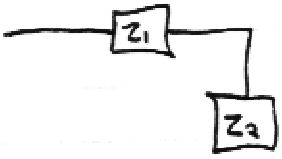
$R_{NEW} = 15915.5\Omega$

~~Rb~~ $R_b = 9319.7\Omega$

3.



$H(s) = \frac{V_o(s)}{V_1(s)}$



$\frac{Z_2}{Z_1+Z_2} = H(s)$ - ONE WAY

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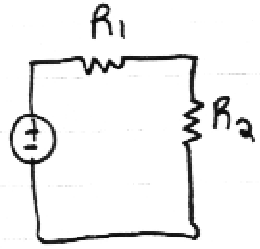

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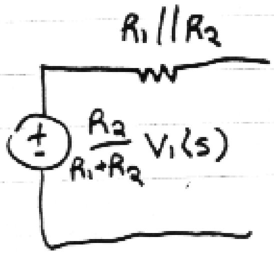

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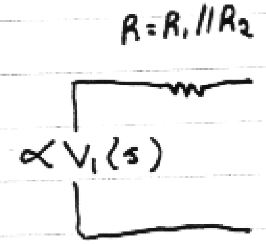

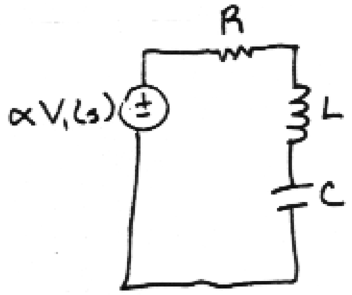

$$\frac{V_o(s)}{\alpha V_1(s)} = \frac{sL + \frac{1}{sC}}{sL + \frac{1}{sC} + R} \qquad \frac{K(s^2 + \omega_0^2)}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

↑

Notch Filter

$K = \alpha = \dfrac{R_2}{R_1+R_2}$ $\quad$ $\omega_0 = \dfrac{1}{\sqrt{LC}}$

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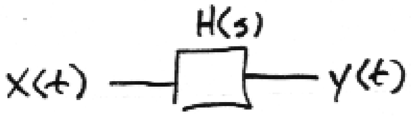
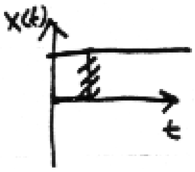


x(t) is a step function



$X(s) = LT\{x(t)\}$

$Y(s) = LT\{y(t)\}$

$\therefore H(s) = \dfrac{Y(s)}{X(s)} \therefore Y(s) = H(s)X(s$

If $x(t) = u(t)$

$X(s) = \dfrac{1}{s}$

$$Y(s) = \frac{2s}{s^2+2s+100}\left(\frac{1}{s}\right) = \frac{2}{s^2+2s+100}$$

$$\therefore y(t) = LT^{-1}\left\{\frac{2}{s^2+2s+100}\right\}$$

$Y(s) = \dfrac{A}{s-P_1} + \dfrac{B}{s-P_2}$ $\quad$ $P_1, P_2$ poles or roots of $s^2+2s+100=0$

$P_1 = -1+j9.95 \quad P_2 = -1-j9.95 = P_1^*$

$$Y(s) = \frac{A}{s-P_1} + \frac{A^*}{s-P_1^*}$$

$$y(t) = LT^{-1}\{Y(s)\} = LT^{-1}\left\{\frac{A}{s-P_1}\right\} + LT^{-1}\left\{\frac{A^*}{s-P_1^*}\right\}$$

$$= \left(Ae^{P_1 t} + A^* e^{P_1^* t}\right)u(t)$$

$$= 2\,\text{real}\{Ae^{P_1 t}\}\,u(t) \qquad A = |A|e^{j\angle A}$$

$2\,\text{real}\{|A|e^{j\measuredangle A}e^{P_1t}\}u(t)$

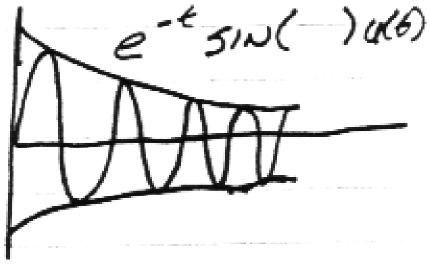
$P_1 = \alpha + j\beta = -1 + j\,9.95$

$y(t) = 2\,\text{real}\{|A|e^{j\{\overset{\beta}{\cancel{\alpha}}t+\measuredangle A\}}e^{\alpha t}\}u(t)$ $\quad (\text{Real}\; e^{jx} = \cos x)$

Note

$= 2e^{\alpha t}|A|\{\text{real}\, e^{j(\beta t+\measuredangle A)}\}u(t)$

$= 2|A|e^{\alpha t}\cos(\beta t + \measuredangle A)\,u(t)$



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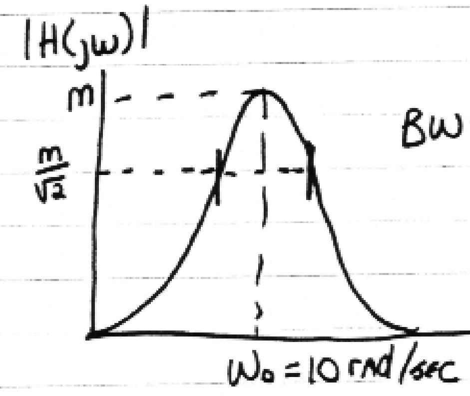
$H(s) = \dfrac{2s}{s^2+2s+100} = \dfrac{Y(s)}{X(s)}$ $\quad s^2Y(s) + 2sY(s) + 100Y(s) = 2sX($

$\dfrac{d^2y(t)}{dt^2} + 2\dfrac{dy(t)}{dt} + 100y(t) = \dfrac{dx(t)}{dt}$



$BW = \dfrac{\omega_0}{Q}$

$H(s) = \dfrac{Ms\frac{\omega_0}{Q}}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$ $\quad 2s = 2s \quad \therefore M = 1$

$M = 1 \quad \omega_0 = 10\,\text{rad/sec} \quad Q = 5$

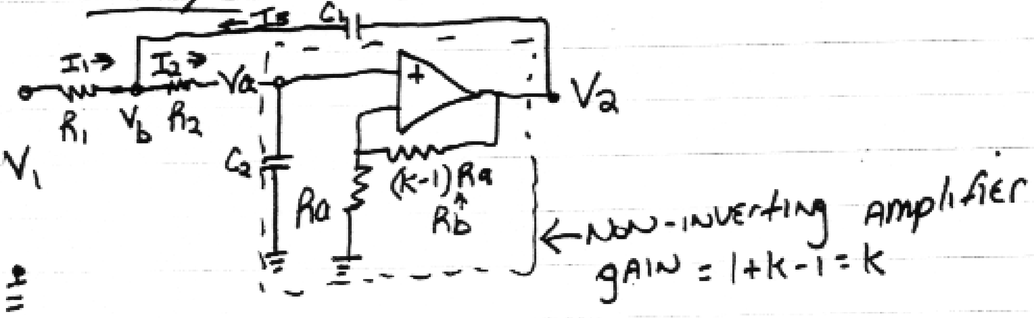
$BW = 2\,\text{rad/sec}$

## $2^{\underline{nd}}$ order Active filters using a single opamp

6/1/98

Low Pass Filter

Example: The Sallen & key LPF.



$$\frac{V_2(s)}{V_1(s)} = H(s) = ?$$

Analysis:

$$\frac{V_2}{V_a} = k \quad \therefore \; V_a = \frac{V_2}{k} \quad ①$$

$$I_1 + I_3 = I_2 \qquad I_1 = \frac{V_1 - V_b}{R_1} \qquad I_3 = \frac{V_2 - V_b}{1/sC_1} \qquad I_2 = V_a s C_2$$

$$\frac{V_1 - V_b}{R_1} + \frac{V_2 - V_b}{1/sC_1} = V_a s C_2 \quad = \quad \frac{V_1 - V_b}{R_1} + (V_2 - V_b)sC_1 = V_a s C_2 \quad ②$$

$$\frac{V_a}{V_b} = \frac{\frac{1}{sC_2}}{\frac{1}{sC_2} + R_2} = \frac{1}{1 + sC_2R_2} = \frac{V_a}{V_b} \quad ③$$

From ③

$$V_b = V_a(1 + sC_2R_3)$$

$$V_b = \frac{V_2}{k}(1 + sC_2R_3)$$

From ① ② + ④

$$\frac{V_2(s)}{V_1(s)} = \frac{K/(R_1R_2C_1C_2)}{s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}-\frac{k}{R_2C_2}\right)s+\frac{1}{R_1R_2C_1C_2}}$$

2nd order LPF

$$\boxed{\frac{K\omega_0^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}}$$

$$\omega_0^2 = \frac{1}{R_1R_2C_1C_2} \qquad \frac{\omega_0}{Q} = \frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}-\frac{K}{R_2C_2}$$

## DESIGN EQUATIONS

$$R_1 = R_2 = R \qquad C_1 = C_2 = C$$

$$R_A = R$$

$$R_b = (K-1)R_A = 3-\frac{1}{Q}-1 = 2-\frac{1}{K} = R_b$$

$$\therefore \omega_0 = \frac{1}{RC}$$

$$\frac{\omega_0}{Q} = \frac{3}{RC}-\frac{K}{RC} = \frac{(3-K)}{RC}$$

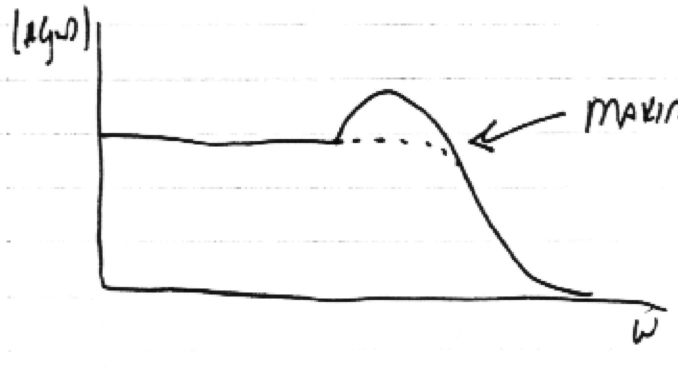
$$\therefore Q = \frac{1}{(3-K)} \qquad \therefore K = 3-\frac{1}{Q}$$

Normalized Circuit

Let $\omega_0 = 1$ rad/sec $\Rightarrow r = 1\Omega \quad C = 1f$

*Example:*

Design a second order LPF to have a maximumally flat mag. response + 3db cutoff freq. = 1kHz given that C's = 0.1uF are there.



maximumally flat ⇒ Q = .707

$W_0 = 2\pi f_0 = 2\pi(1k) = 2\pi \times 10^3$ rad/sec.

$C_{NEW} = 0.1 uF$ $\quad C_{NEW} = \frac{C_{OLD}}{K_f K_m} = \frac{1}{K_m(2\pi \times 10^3)} \Rightarrow K_m = 1.5915 \times 10^3$

$R_{NEW} = K_m R_{OLD} = 1.5915 \times 10^3 \Omega$ $\quad C = .1uF$

$R_a = 1.5915 \times 10^3 \Omega$

$\uparrow$ $R_b = 795.7 \Omega$ $\quad \uparrow$

Plug VALUES into old circuit.

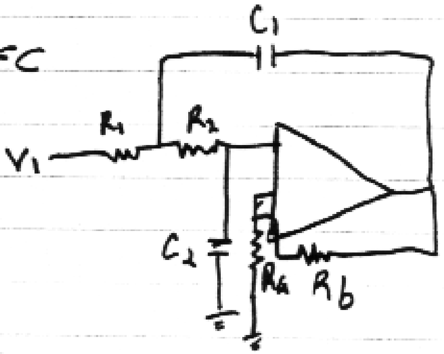
$K = 1.58$

## DESIGN #2

$W_0 = 1$ rad/sec

$K = 1$

$R_1 = R_2 = 1\Omega$



$W_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$

$\frac{W_0}{Q} = \frac{1}{R_2 C_1} + \frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} - \frac{\overset{1}{\cancel{K}}}{R_2 C_2}$

$Q = \frac{C_1}{2} \quad C_1 = 2Q$

$\frac{W_0}{Q} = \frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} = \frac{1}{Q}$

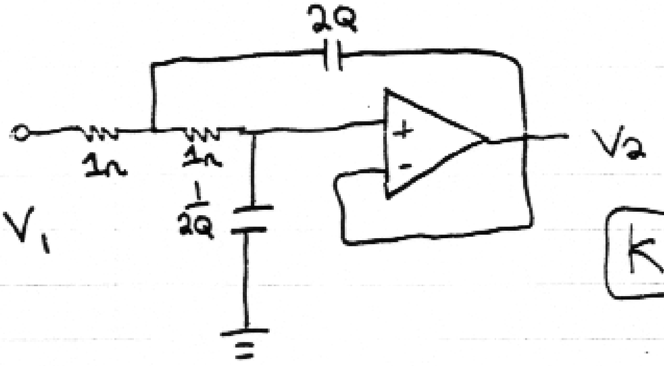
$\Downarrow$

$\frac{2}{C_1}$

$C_1 C_2 = 1$

$\therefore C_2 = \frac{1}{2Q}$

$W_0 = \frac{1}{\sqrt{C_1 C_2}} = 1$



$$\frac{C_1}{C_2} = 4Q^2$$

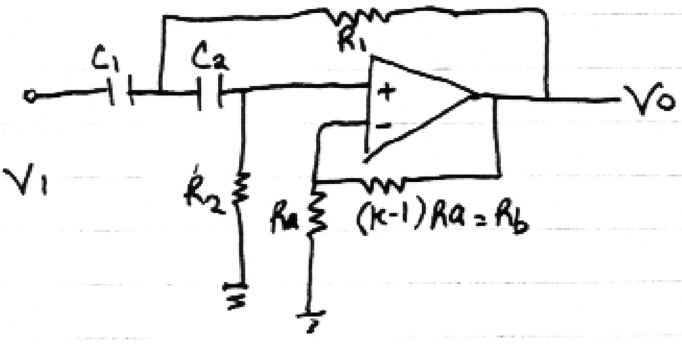
$$\boxed{K=1}$$

## DESIGN #3 $\omega_0 = 1\ rad/sec$

$$\frac{C_1}{C_2} = Q \qquad \frac{R_2}{R_1} = Q \qquad H_a(s) = \frac{V_2(s)}{V_1(s)} = \frac{2}{s^2 + \frac{s}{Q} + 1}$$

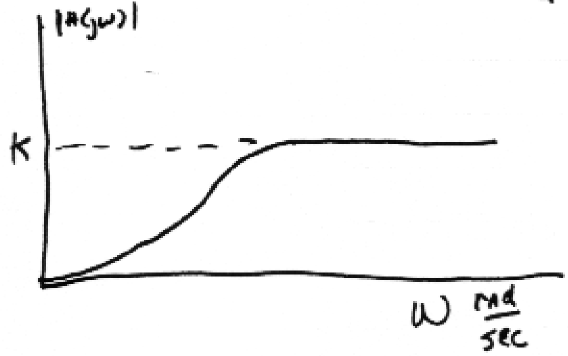
---

## SAlken &Key HPF Design



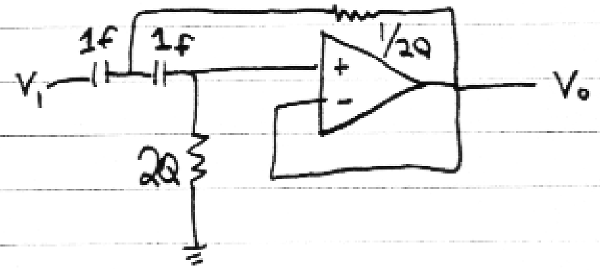
DESIGN #1: Equal R + Equal C $\omega_0 = 1\ ^{rad}/_{sec}$

$R_2 = 1\Omega$ $R_a = 1\Omega$ $R_b = 2 - \frac{1}{Q}\Omega$ $C_1 = C_2 = 1f$ $R_1 = 1\Omega$



Design #2

Unity gain (k=1)



## Design #3

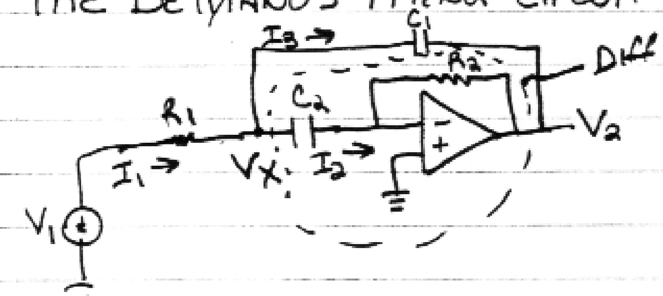
$R_1 = 1\Omega \quad K=2 \quad C_1 = 1f \quad C_2 = \frac{1}{Q} f \quad R_2 = Q\Omega$

$R_A = R_b = 1\Omega$

---

## BandPass Filter

The Delyianus-Friend Circuit



$$\frac{V_2(s)}{V_1(s)} = H(s)$$

$$\frac{V_2}{V_X} = -sC_2R_2 \quad (-1) \qquad V_X = \frac{-V_2}{sC_2R_2} \quad (-1)$$

KCL to Node X

$$I_1 = I_2 + I_3 \qquad \frac{V_1 - V_X}{R_1} = \frac{V_X sC_2}{1} + (V_X - V_2)sC_1 \quad (-2)$$

$$\frac{V_2(s)}{V_1(s)} = \frac{-\frac{1}{R_1C_1}(s)}{s^2 + \frac{1}{R_2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)s + \frac{1}{R_1R_2C_1C_2}}$$

$$C_1 = C_2 = C \qquad \omega_0^2 = \frac{1}{R_1 R_2 C_1 C_2} \Rightarrow \omega_0 = \frac{1}{C\sqrt{R_1 R_2}}$$

$$\frac{\omega_0}{Q} = \frac{2}{R_2 C} \qquad Q = \frac{1}{\dfrac{\cancel{C}\sqrt{R_1 \cancel{R_2}}}{\dfrac{2}{\sqrt{R_2}\cancel{C}}}} \Rightarrow Q = \frac{\sqrt{R_2}}{2\sqrt{R_1}}$$

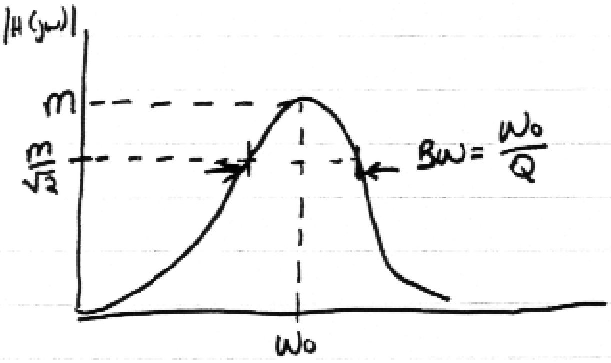
Let $\omega_0 = 1 \frac{rad}{sec}$ ~~[illegible]~~ $1 = \frac{1}{C\sqrt{R_1 R_2}} \therefore C = \frac{1}{\sqrt{R_1 R_2}}$

$R_1 = \underline{1}\,\Omega \therefore R_2 = 4Q^2\,\Omega \therefore C = \frac{1}{2Q}\,\Omega$

Normalized Circuit $\Rightarrow \omega_0 = 1 \frac{rad}{sec}$

$$H(s) = \frac{-m\frac{\omega_0}{Q}s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

Inverting BPF



$Q > 1$

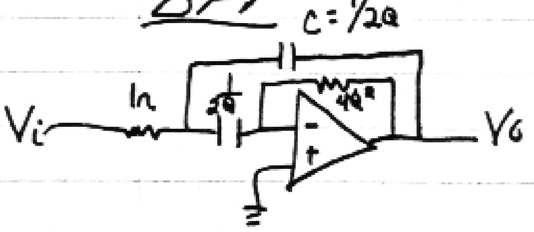
$$H(s) = \frac{-\frac{1}{R_1 C_1}s}{s^2 + \frac{2}{R_2 C}s + \frac{1}{R_1 R_2 C^2}} \qquad m\frac{\omega_0}{Q} = \frac{\omega_0}{Q} \qquad \frac{-1}{R_1 C_1} = \frac{2}{R_2 C}$$

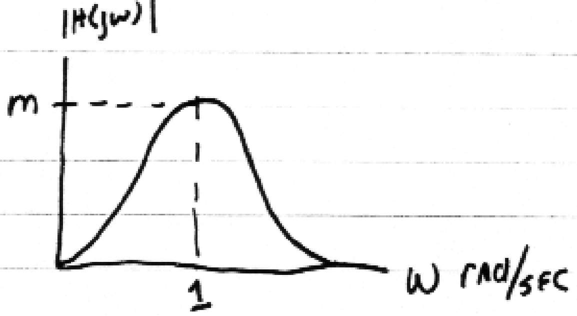
$$m - R_2 \cancel{C} = 2R_1 \cancel{C} \qquad m = \frac{R_2}{2R_1}$$

$$\text{But } Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}} \qquad \therefore m = 2Q^2$$

6/3/98

## BPF



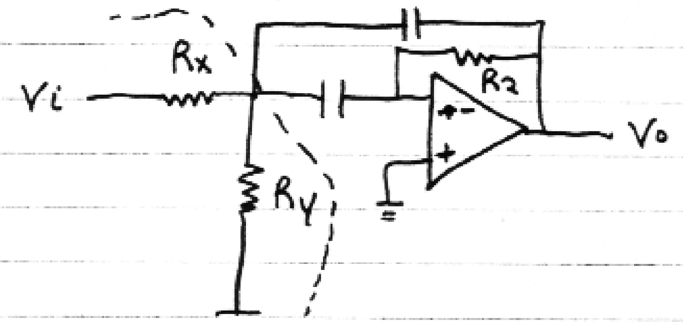


$\omega_0 = 1$ rad/sec

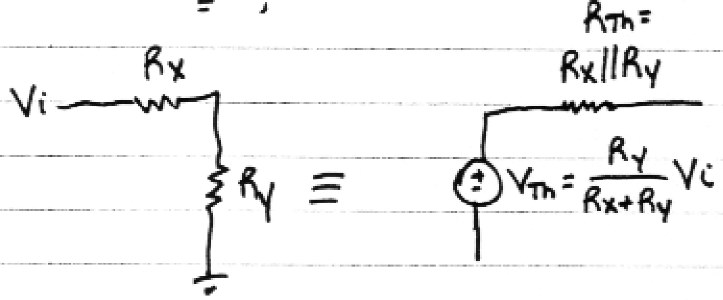
$$H(s) = \frac{-\frac{m}{Q}s}{s^2+\frac{s}{Q}+1}$$

$m = 2Q^2$

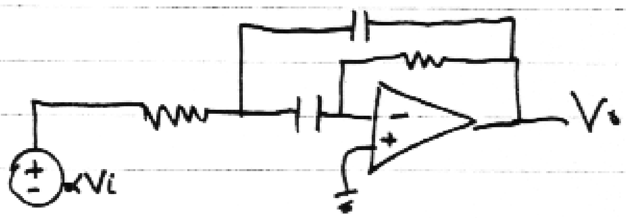
## Modified Circuit



$m = 1$



$\alpha = \frac{R_y}{R_x+R_y}$



$$\frac{V_o}{\alpha V_i} = \frac{-\frac{2Q^{\cancel{2}}}{\cancel{Q}}s}{s^2+\frac{s}{Q}+1}$$

$$= \frac{-2Qs}{s^2+\frac{s}{Q}+1}$$

$$R_{Th} = 1\Omega \quad \frac{R_xR_y}{R_x+R_y} = 1 \quad ①$$

$$\frac{V_o}{V_i} = \frac{-\alpha 2Qs}{s^2+\frac{s}{Q}+1}$$

At $\omega_0 = 1$ $\quad \left| \frac{V_0}{V_1} \right| = \alpha 2Q^2 = 1$

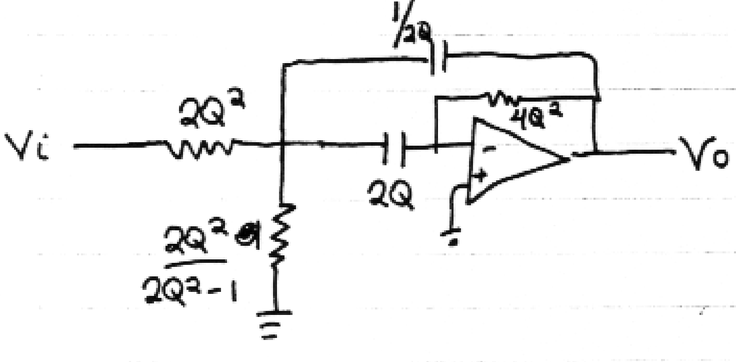
$$\therefore \alpha = \frac{1}{2Q^2} = \frac{R_Y}{R_X + R_Y} \quad ②$$

- USING 1 & 2 WE FIND $R_X$ & $R_Y$ -

$\boxed{\text{From 2}}$ $\quad \frac{1}{2Q^2} = \frac{1}{1 + \frac{R_X}{R_Y}}$ $\qquad$ FIND $\frac{R_X}{R_Y}$

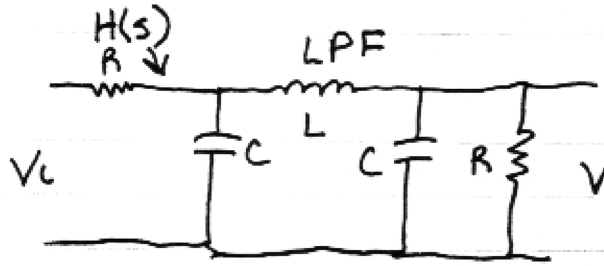
$\boxed{\text{From 1}}$ $\quad \frac{R_X}{1 + \frac{R_X}{R_Y}} = 1 \Rightarrow$ GET $R_X$

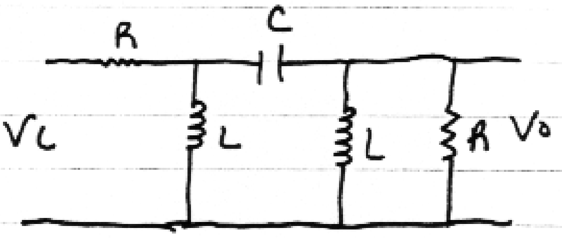
$$\therefore R_X = 2Q^2 \qquad R_Y = \frac{2Q^2}{2Q^2 - 1}$$



$$H(s) = \frac{-s/Q}{s^2 + s/Q + 1}$$

# Filter Transformation





Transformation ↘ $G(s) = H(s)\Big|_{s=\frac{\omega_0}{s}}$

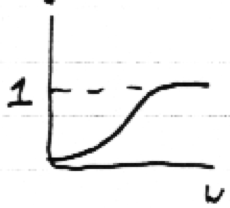
## Example

1st order LPF

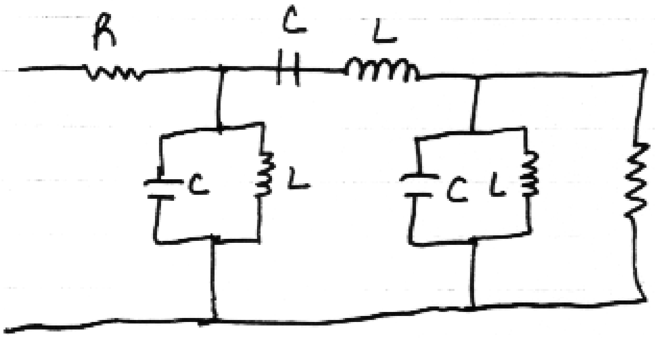
$H(s) = \dfrac{1}{s+1}$

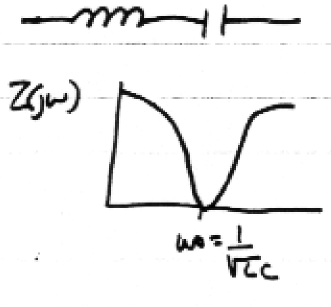
1st order HPF

$G(s) = H(s)\Big|_{s=1/s} = \dfrac{1}{\frac{1}{s}+1} = \dfrac{s}{s+1}$

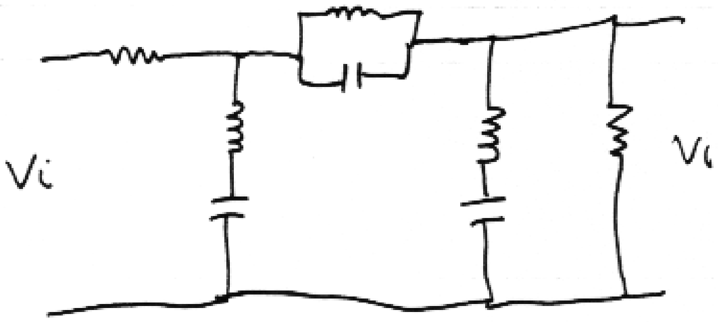


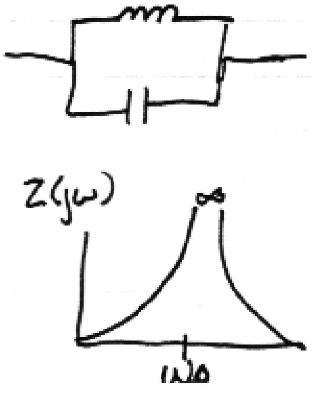
BPF





Notch

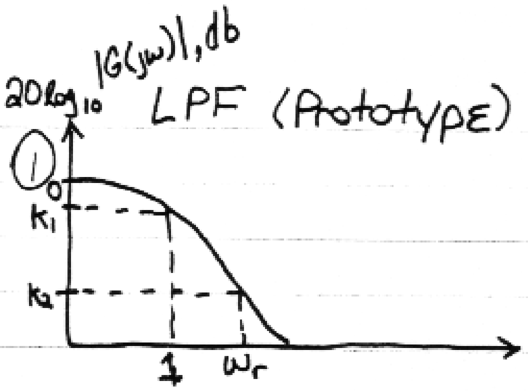




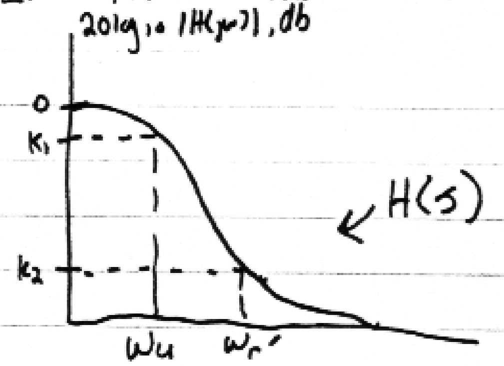
$Z(j\omega) = \dfrac{1}{Y(j\omega)}$

$= \dfrac{1}{j\omega C + \frac{1}{j\omega L}}$

# LPF (Prototype) to LPF Transformation



G(s) = Transfer function



$$H(s) = G(s)\Big|_{s=\frac{s}{\omega_u}}$$

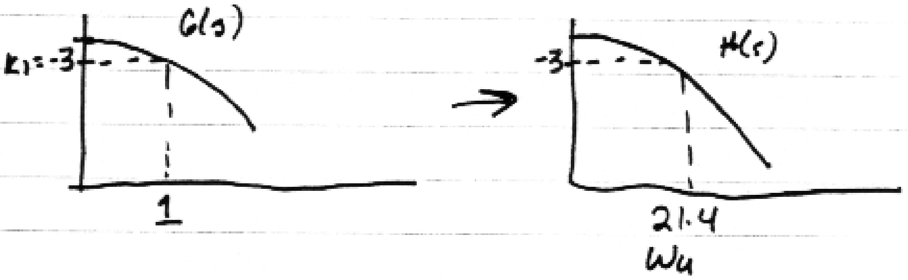
## Design Equations

$\omega_r' = \omega_u \omega_r$

$\omega_r = \dfrac{\omega_r'}{\omega_u}$

Given: Normalized

$$\frac{1}{(s^2+.765s+1)(s^2+1.85s+1)}$$



$$H(s) = G(s)\Big|_{s=s/\omega_u \Rightarrow s=s/21.4}$$

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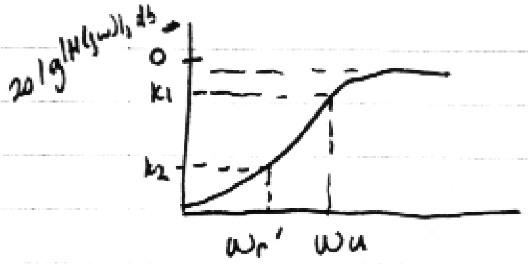
LPF → HPF



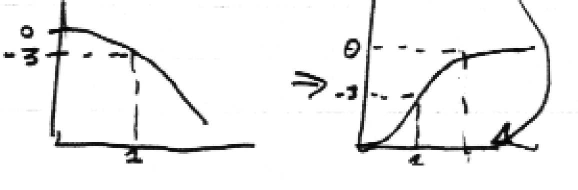
$$H(s) = G(s)\Big|_{s=\frac{\omega_u}{s}}$$

## Design Equations:

$\omega_r' = \dfrac{\omega_u}{\omega_r}$

$\omega_r = \dfrac{\omega_u}{\omega_r'}$

Ex| $G(s) = \dfrac{1}{s^3+2s^2+2s+1}$

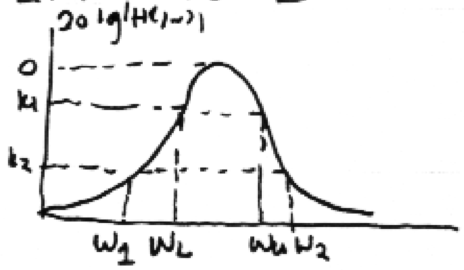


$H(s) = G(s)\Big|_{s=\frac{\omega_u}{s}}$

$= \frac{1}{s}$

...

## ③ LPF to BPF



$$H(s) = G(s)\Big|_{s = \frac{s^2 + W_l W_u}{s(W_u - W_L)}} = \frac{s^2 + W_0^2}{s\,bW}$$

$$W_0 = \sqrt{W_l W_u} \qquad bW = W_u - W_L$$

### Design Equations

$W_V = 2$ $W_{AV} =$ [illegible] $W_C = Q$

$$W_{AV} = (W_u - W_L)/2$$

$W_r = 1$

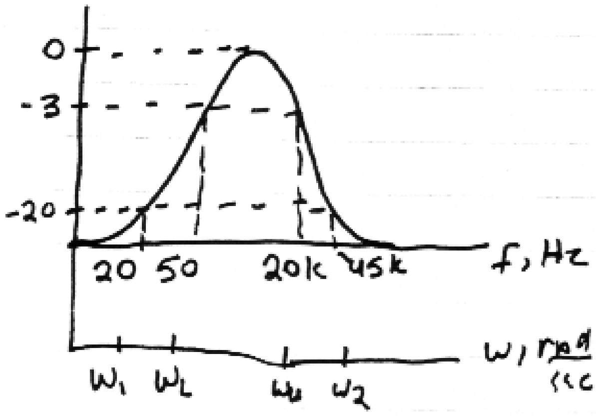
$$W_1 = (W_V^2 W_{AV}^2 + W_u W_L)^{1/2} - W_{AV} W_r$$

$$W_2 = (W_V^2 W_{AV}^2 + W_L W_u)^{1/2} + W_{AV} W_r$$

$W_r = \min\{|A|, |B|\}$ - minimum of either

where $A = \dfrac{-W_1^2 + W_L W_u}{W_1(W_u - W_L)}$ + $B = \dfrac{W_2^2 - W_L W_u}{W_2(W_u - W_L)}$

### Example:



$W_1 = 2\pi(20)$

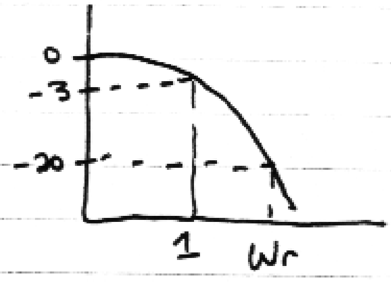
$W_L = 2\pi(50)$

$W_u = 2\pi(20k)$

$W_2 = 2\pi(45k)$

BPF → LPF - Backward Design Eq.

Continued!



Design the prototype LPF

Ex/ Use a butterworth Approx. N=3

$$G(s) = \frac{1}{s^2+2s^2+2s+1}$$

$W_r = 2.254$ rad/sec

→ Apply LPF to BPF Transformation

$$H(s) = G(s)\Big|_{s=\frac{s^2+W_uW_L}{s(W_u-W_L)}} = \frac{s^2+3.94784\times10^7}{s(1.25349\times10^5)}$$

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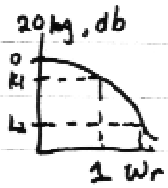
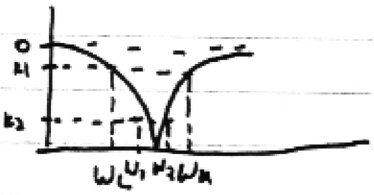
④ LPF to Notch X-formation





$$H(s) = G(s)\Big|_{s=\frac{s(W_u-W_L)}{s^2+W_LW_u}}$$

Design Equations

Fow/ $W_{AV} = (W_u - W_L)/2$

$$W_1 = \left[(W_{AV}/W_r)^2 + W_LW_u\right]^{1/2} - \frac{W_{AV}}{W_r}$$

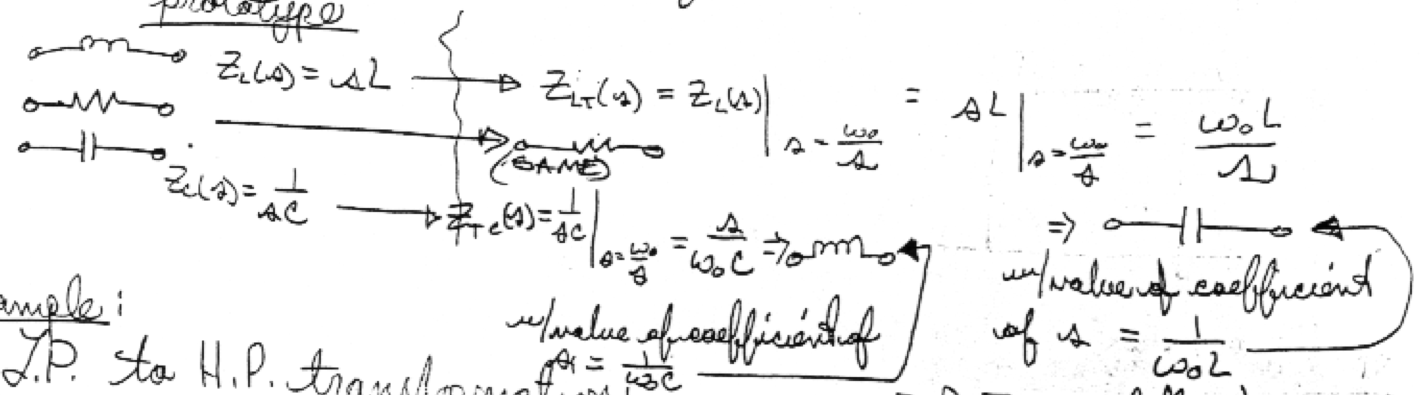
$$W_2 = \left[(W_{AV}/W_r)^2 + W_LW_u\right]^{1/2} + \frac{W_{AV}}{W_r}$$

Back/ $W_r = \min\{|A|, |B|\}$

$$A = \frac{W_1(W_u-W_L)}{-W_1^2+W_LW_u}$$

$$B = \frac{W_2(W_u-W_L)}{-W_2^2+W_LW_u}$$

## Applying the transformation directly on the circuit elements.

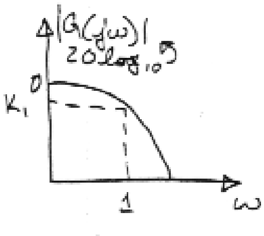


$Z_L(s) = sL \longrightarrow Z_{LT}(s) = Z_L(s)\Big|_{s=\frac{\omega_0}{s}} = sL\Big|_{s=\frac{\omega_0}{s}} = \frac{\omega_0 L}{s}$ ⇒ w/ value of coefficient of $s = \frac{1}{\omega_0 L}$

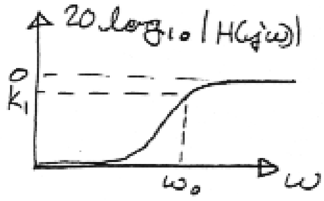
$Z_C(s) = \frac{1}{sC} \longrightarrow Z_{TC}(s) = \frac{1}{sC}\Big|_{s=\frac{\omega_0}{s}} = \frac{s}{\omega_0 C}$ ⇒ w/ value of coefficient of $s = \frac{1}{\omega_0 C}$

Example:

### L.P. to H.P. transformation:

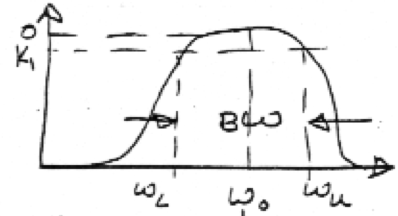


$G(s)$



$H(s) = G(s)\Big|_{s=\frac{\omega_0}{s}}$

### B.P.F. for ladder:



$H(s) = G(s)\Big|_{s=\frac{s^2+\omega_0^2}{s(bw)}}$

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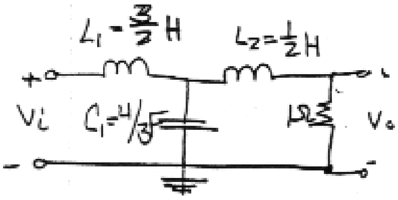
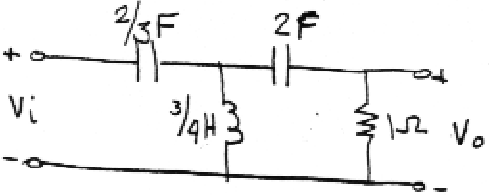
Example: Transform the given prototype 3rd order L.P.F. into a prototype 3rd order H.P.F.



(Prototype L.P.F.)

H.P.F. w/ $\omega_0 = 1$ rad/sec

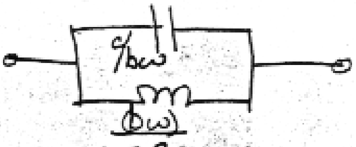


---

Example: Transform L.P.F. to B.P.F.

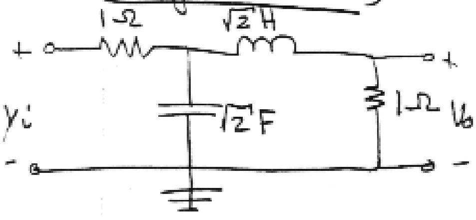
$(s) \to Z_{LT}(s) = sL\Big|_{s=\frac{s^2+\omega_0^2}{s(bw)}} = \frac{s^2+\omega_0^2}{s(bw)}L = \frac{L}{bw}s + \frac{\omega_0^2 L}{bw\, s} =$

$(s) \to Z_{TC}(s) = \frac{1}{sC}\Big|_{s=\frac{s^2+\omega_0^2}{s(bw)}} = \frac{s(bw)}{(s^2+\omega_0^2)C} = Y_{TC}(s) = \frac{1}{Z_{TC}(s)} = \frac{(s^2+\omega_0^2)C}{s(bw)} = \frac{sC}{(bw)} + \frac{\omega_0^2 C}{s(bw)}$



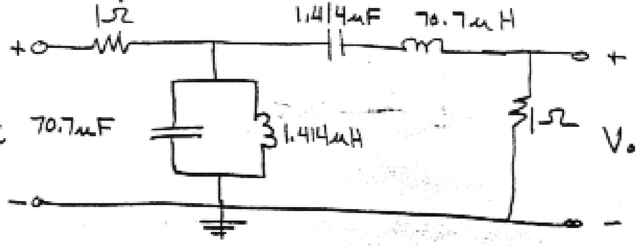
xample: Given the prototype L.P.F. (normalized) circuit of a 2nd order Butterworth L.P.F., apply a L.P. to B.P. transformation h that $\omega_o(BP) = 100k\ ^{rad}/_{sec}$ and $bw = 20k\ ^{rad}/_{sec}$. (2)

Original L.P.F.



(prototype L.P.F.)

Transformed filter (B.P.F.)



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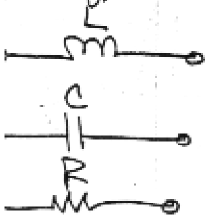
## P. to Band stop (notch) transformation:

otatype circuit composed of:

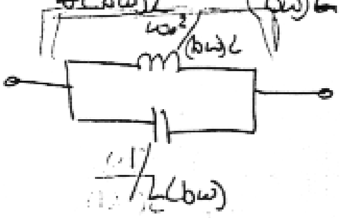


Transformed circuit components:

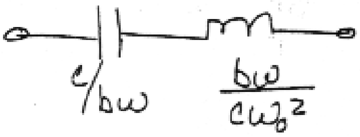
$Z_L(s) = sL \Rightarrow Z_{LT}(s)\Big|_{s=\frac{s(bw)}{s^2+\omega_o^2}} \Rightarrow Z_{LT}(s) = \frac{s(bw)L}{s^2+\omega_o^2}$

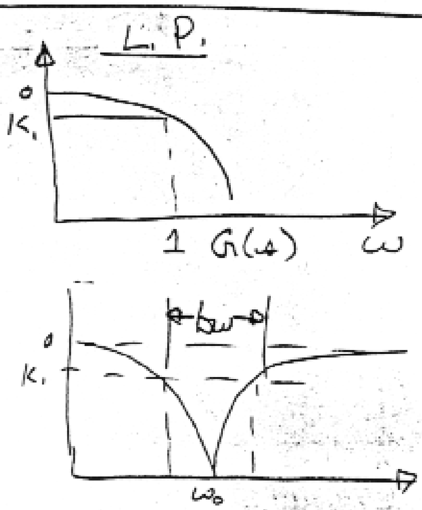
$\Rightarrow Y_{LT}(s) = \frac{s^2+\omega_o^2}{s(bw)L} = \frac{s}{(bw)L} + \frac{\omega_o^2}{s(bw)L}$

$\Rightarrow Y_L =$



$s) = \frac{1}{sC} \Rightarrow Z_{TC}(s)\Big|_{s=\frac{s(bw)}{s^2+\omega_o^2}} \Rightarrow$

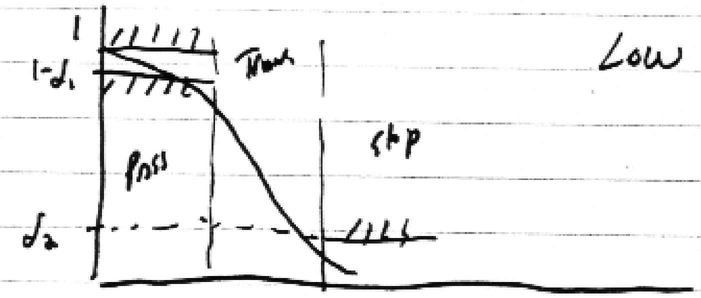




$H(s) = G(s)\Big|_{s=\frac{s(bw)}{s^2+\omega_o^2}}$

# Filter Design

## Butterworth Approx



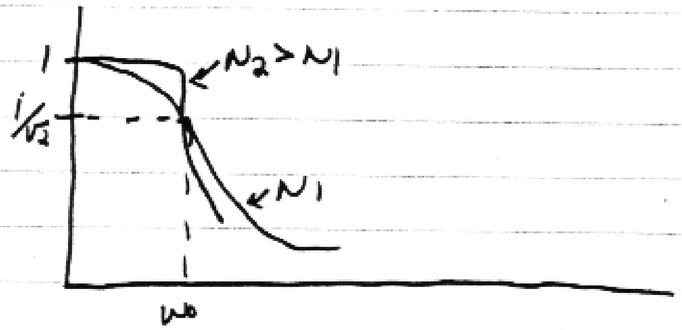
Low Pass Filter

$$|H_N(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^{2N}}}$$

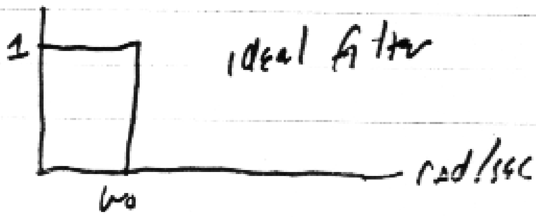
$\downarrow$ order

N : order

$W_0$ = 3dB cutoff or bandwidth



As $N \to \infty$



$W < W_0$

$|H_N(j\omega)| = 1$

$N \to \infty$

$W > W_0$

$H_N(j\omega) = \leftarrow$

1- $H_N(j0)| = 1$ for All N

2- $H_N(j\omega_0) = 1/\sqrt{2}$

3- $$\left.\frac{d^k|H_N(j\omega)|}{d\omega_k}\right|_{\omega=0} = 0$$

for $k = 1, 2, 3, \ldots 2N-1$

Maximumally flat response

## Finding the transfer function $H_N(s)$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^{2N}}}$$

$$\boxed{\frac{1}{s^2+\frac{s}{Q_1}+1}}$$

$$H_N(s)H_N(-s) = |H(j\omega)|^2\Big|_{\omega=s/j}$$

$$= \frac{1}{1+\left(\frac{\omega}{\omega_0}\right)^{2N}}\Bigg|_{\omega=s/j}$$

$$= \frac{1}{1+\left(\frac{s/j}{\omega_0}\right)^{2N}}$$

Consider the normalized case
∴ $\omega_0 = 1$ rad/sec

$$H(s) = \frac{1}{s+1}$$

$$H(j\omega) = H(s)\Big|_{s=j}$$

$$= \frac{1}{1+j\omega}$$

$$|H(j\omega)|^2 = H(j\omega)H^*(j\omega$$

$$= H(j\omega)H(-j\omega)$$

$$= H(s)H(-s)\Big|_{s=j\omega}$$

$$\therefore H(s)H(-s) = |H(j\omega)|^2\Big|_{\omega=s/j}$$

$$\therefore H_N(s)H_N(-s) = \frac{1}{1+(-1)^N s^{2N}} \Rightarrow j^{2N} = (j^2)^N = (-1)^N \leftarrow$$

## Find the poles:

$1+(-1)^N s^{2N} = 0 \quad N=1 \quad 1+(-1)s^2 = 1-s^2 \quad s=\pm 1$

$\therefore H_1(s)H_1(-s)$

$\left(\frac{1}{s+1}\right)\cancel{\left(\frac{1}{s-1}\right)}$ not stable $\qquad H_1(s) = \left(\frac{1}{s+1}\right)$

$1+(-1)^N s^{2N} = 0 \quad N=2 \quad 1+(1)s^4 = 1+s^4 = 0 \Rightarrow s^4 = -1$

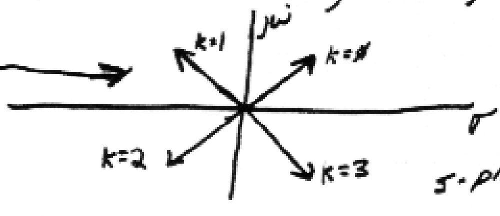
$= 1e^{j\pi}$

$= e^{j(\pi+2k\pi)}$

$k=0,1,2,3$

$s_4 = e^{j(\pi+2k\pi)/4}, \; k=0,1,2,3$

All have mag=1



s-plane

$k=0 \quad s=e^{j\pi/4}$

$k=1 \quad s=e^{j(\pi/4+\pi/2)}$

$k=1$ & $k=2$ are stable

$$H(s) = \frac{1}{(s-e^{j3\pi/4})(s-e^{-j3\pi/4})} \qquad e^{j\frac{3\pi}{4}} = -0.707 + j.707$$

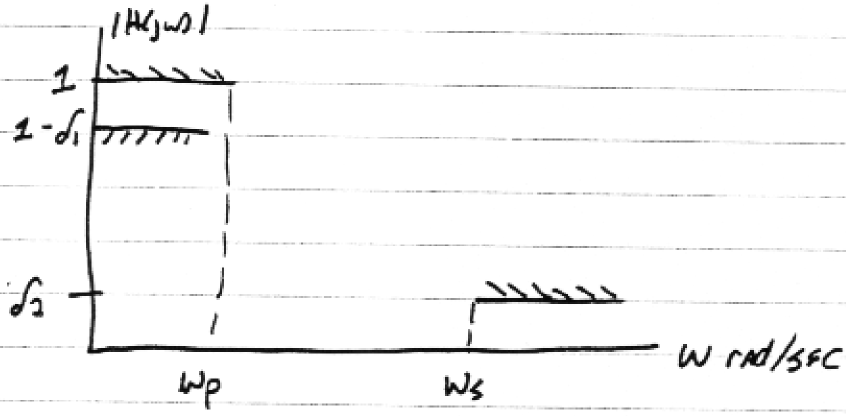
$$= \frac{1}{s^2+\sqrt{2}s+1} \qquad \sqrt{2} \text{ is from } \quad 2 \cdot \frac{1}{\sqrt{2}} = \sqrt{2}$$

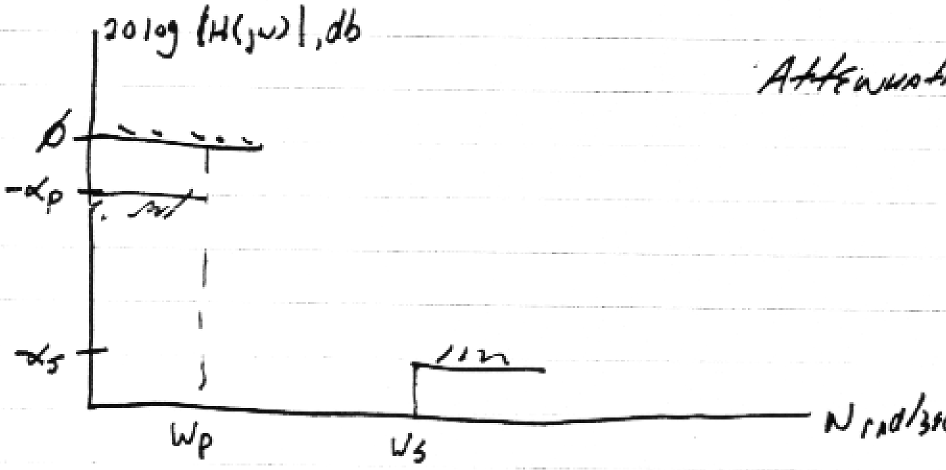
If N=3

$$\boxed{H_3(s) = \frac{1}{s+1} \cdot \frac{1}{s^2+s+1}}$$

All poles are on page 165-166 in book.

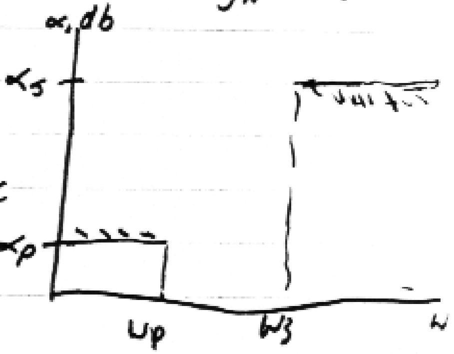
Design Equations





$$\text{Attenuation in db} = 20\log_{10}\left|\frac{V_1}{V_0}\right|$$

$$= -20\log_{10}|H(j\omega)|$$



Need to find $N = ?$ ← order

$\omega_0 = ?$ ← cutoff

$$\alpha = -20\log_{10}|H_N(j\omega)| = -20\log_{10}\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^{2N}}}$$

$$\alpha = 10\log_{10}\left\{1+\left(\frac{\omega}{\omega_0}\right)^{2N}\right\}$$

---

$\rightarrow \alpha = \alpha_p \; @ \; \omega = \omega_p \leftarrow$

$$\alpha_p = 10\log_{10}\left\{1+\left(\frac{\omega_p}{\omega_0}\right)^{2N}\right\} \quad \text{①}$$

$$\frac{\alpha}{10} = \log_{10}\left\{1+\left(\frac{\omega}{\omega_0}\right)^{2N}\right\}$$

$$10^{\alpha/10} = 1+\left(\frac{\overset{\text{nothing}}{\omega_R}}{\omega_0}\right)^{2N}$$

$$\left[\omega_0 = \frac{\omega}{\left[10^{\alpha/10}-1\right]^{\frac{1}{2N}}}\right] \text{ General}$$

$\left[\begin{array}{l}\text{Ceil} \rightarrow \infty \\ \text{fix} \rightarrow -\infty \\ \text{round} \rightarrow \end{array}\right]$ Matlab

$$\omega_0 = \frac{\omega_p}{\left[10^{\alpha_p/10}-1\right]^{1/2N}} \quad \text{①} \text{ Rewritten}$$

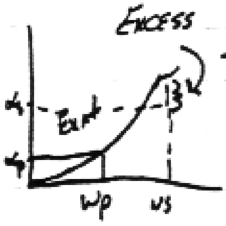
$$\boxed{N = \frac{\log\left\{\frac{10^{\alpha_s/10}-1}{10^{\alpha_p/10}-1}\right\}}{2\log\frac{\omega_s}{\omega_p}}}$$

Ex/ $N = 2..$

$\therefore N = 3$

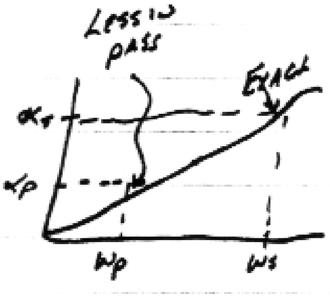
$\rightarrow \alpha = \alpha_s \; @ \; \omega = \omega_s \leftarrow$

$$\omega_0 = \frac{\omega_s}{\left[10^{\alpha_s/10}-1\right]^{1/2N}} \quad \text{②}$$



TAKE ① to find $\omega_0$

TAKE ② to find $\omega_0$



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Example: Design Butterworth LPF with $\omega_p = 1000$ rad/sec $\alpha_p \leq .5$ db
$\omega_s = 2000$ rad/sec $\alpha_s = 20$ db

Find N of filter using Eq. $\boxed{N=5}$

Find $\omega_0$ using Eq. ① $\omega_0 = 1263.2$ rad/sec.
Eq. ② $\omega_0 = 1234$ rad/sec

Realizing the filters

## Active

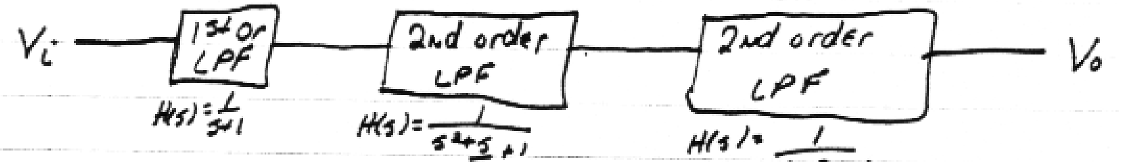
Use table pg. 166 for N odd (normalized) $(\omega_0 = 1)$

$$H(s) = \frac{1}{(s+1)(s^2+\frac{s}{Q_1}+1)(s^2+\frac{s}{Q_2}+1)}$$

for N EVEN

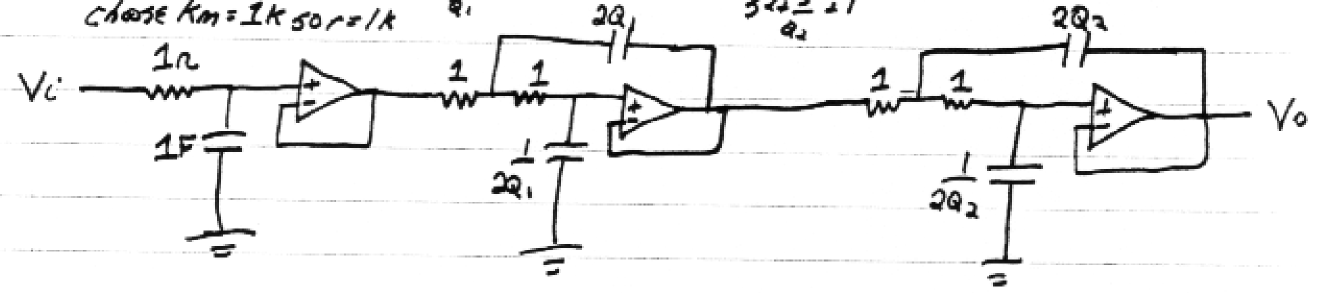
$$H(s) = \frac{1}{(s^2+\frac{s}{Q_1}+1)(s^2+\frac{s}{Q_2}+1)}$$

normalized $$H_5(s) = \left(\frac{1}{s+1}\right)\left(\frac{1}{s^2+\frac{s}{0.62}+1}\right)\left(\frac{1}{s^2+\frac{s}{1.62}+1}\right)$$ where $\omega_0 = 1$ rad/sec



$Q_1 = .62$
$Q_2 = 1.62$

choose Km = 1k so r = 1k



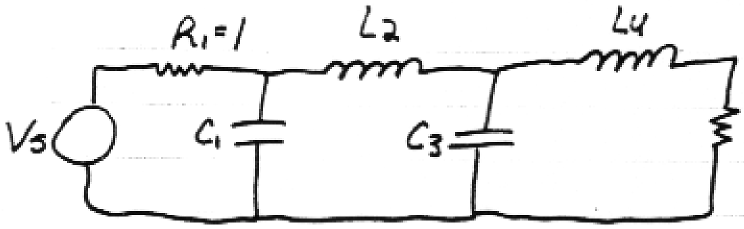
Now we need to frequency scale

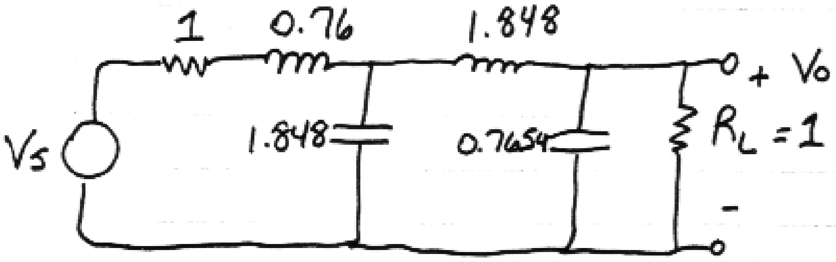
$K_f = \omega_0 = 1234$ rad/sec or 1263.2 rad/sec
for all stages

Example: 6/12/98

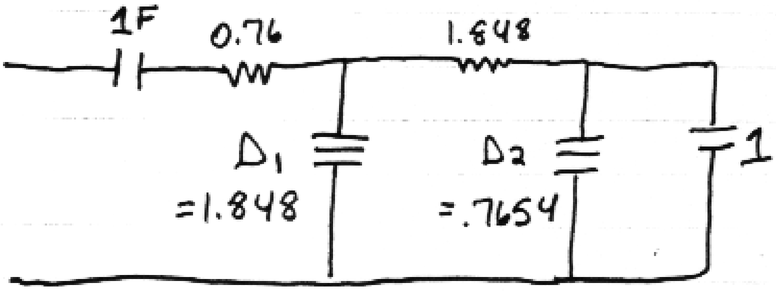
Design a fourth order LPF with $f_o$ = 1kHz

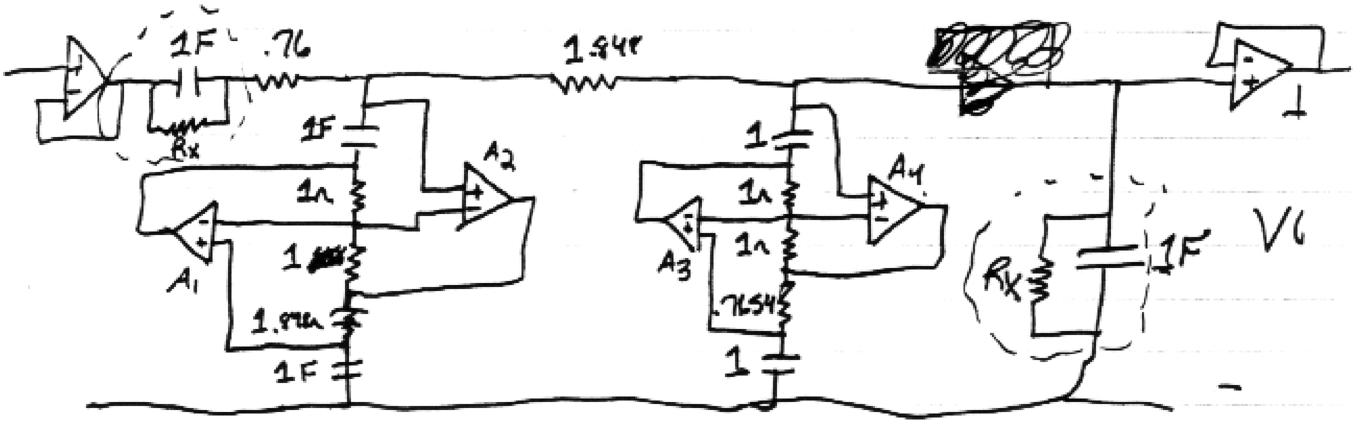
Use a butterworth Approximation:





Bruton's transformation



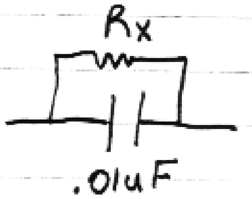


$Kf = \frac{2\pi f_c}{1} = 2\pi \times 10^3$ $C = 0.01uF$

$.01 \times 10^{-6} = \frac{1}{k_f k_m}$ $\therefore K_m = 1.59 \times 10^4$

→ OPAMPS MUST HAVE DC PATH to ground. ←

$A_2$ & $A_4$ are a problem - Add Rx on circuit as shown



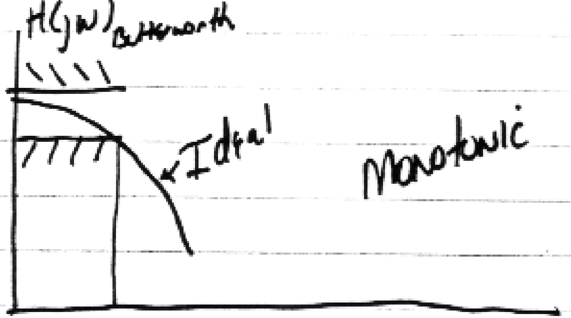
$Rx = 10\left(\frac{1}{\omega_{MIN} C}\right)$ Choose $\omega_{MIN} = 100Hz$

$$10\left(\frac{1}{2\pi(100)(.01uF)}\right)$$

Add a buffer @ input AND ~~[illegible]~~ @ output

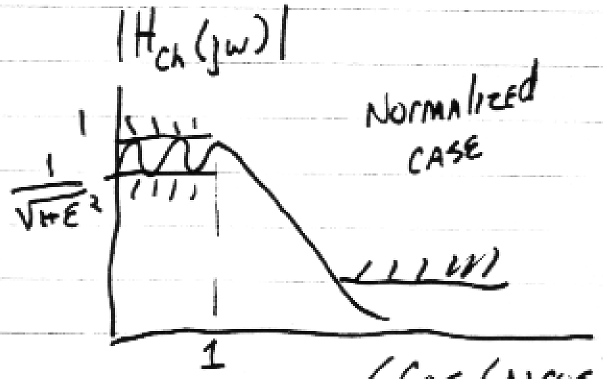
{ PASSIVE HPF → Active
{ PASSIVE LPF → Active

# The Chebyshev Approx



Monotonic

$$|H(jw)| = \frac{1}{\sqrt{1+\left(\frac{w}{w_c}\right)^{2N}}}$$



Normalized CASE

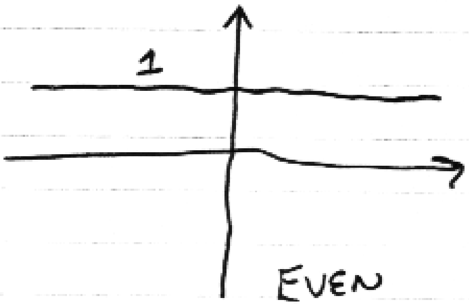
$$|H_{ch}(jw)| = \frac{1}{\sqrt{1+\varepsilon^2 C_N^2(\omega)}}$$

$C_N(\omega)$ = Chebyshev poly of order N.

$$C_N(\omega) = \begin{cases} \cos(N\cos^{-1}\omega) & 0 \le \omega \le 1 \\ \cosh(N\cosh^{-1}\omega) & \omega > 1 \end{cases}$$

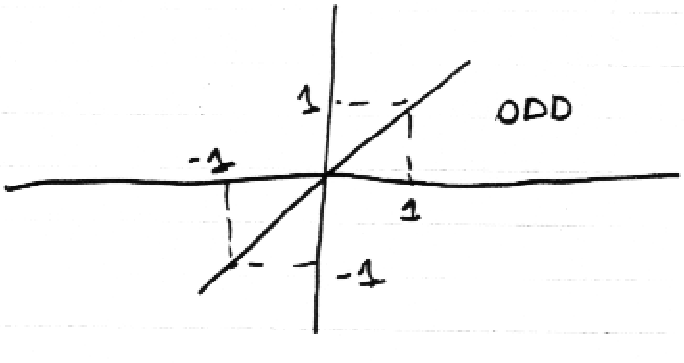
Properties of the chebyshev polynomial

$C_0(\omega) = \cos(0) = 1$



N = 1

$C_1(w) = \cos(\cos^{-1}\omega) = \omega$



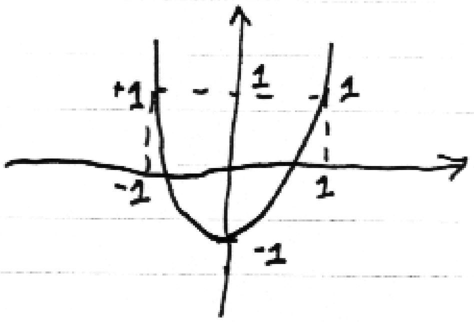
$N = 2$

$C_2(\omega) = \cos(2\cos^{-1}\omega)$

Let $x = \cos^{-1}\omega \Rightarrow \omega = \cos x$

$C_2(\omega) = \cos(2x) = 2\cos^2 x - 1$

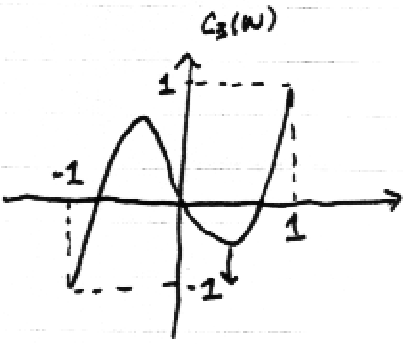
$= 2\omega^2 - 1$



$C_{N+1}(\omega) = 2\omega\, C_N(\omega) - C_{N-1}(\omega)$

$C_3(\omega) = 2\omega\, C_2(\omega) - C_1(\omega)$

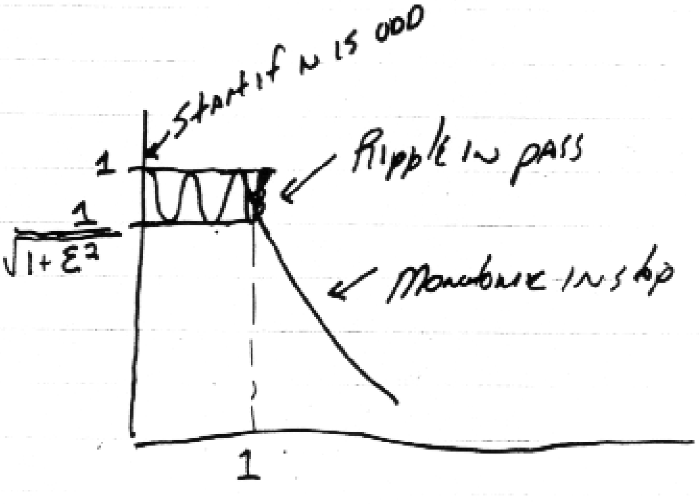
$= 2\omega\{2\omega^2 - 1\} - \omega = 4\omega^3 - 3\omega$



$\frac{dC_3(\omega)}{d\omega} = 12\omega^2 - 3 \Rightarrow \omega^2 = \frac{1}{4} \quad \omega = \pm 1/2$

$C_3(\frac{1}{2}) = -\frac{3}{2} + \frac{4}{8} = -1$

$|H_a(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2 C_N^2(\omega)}}$



$N \equiv$ order

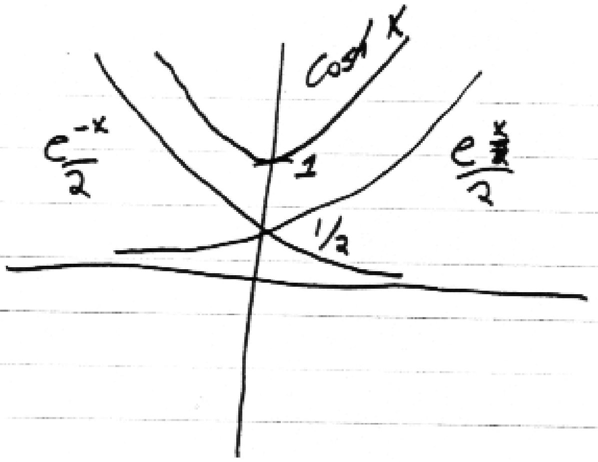
$\epsilon \equiv$ Ripple

# REVIEW

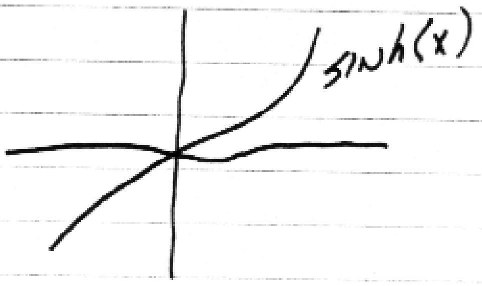
$e^{jx} = \cos x + j\sin x$

$\cos x = \frac{1}{2}\left\{e^{jx} + e^{-jx}\right\}$

$\sin x = \frac{1}{2j}\left\{e^{jx} - e^{-jx}\right.$



$\cosh x = \frac{e^{x} + e^{-x}}{2} = \cos(jx)$



$\sinh x = \frac{e^{x} - e^{-x}}{2}$

$\sin(jx) = \frac{1}{2j}\left\{e^{-x} - e^{x}\right\}$

$= \frac{j}{2}\left\{e^{x} - e^{-x}\right\} = j\sinh(x)$

---

$C_N(\omega) = \cos(N\cos^{-1}\omega)$

If $\omega > 1$

$\cos^{-1}(\omega) = x$

$\cos x = \omega \Leftarrow$ real

$\frac{e^{jx} + e^{-jx}}{2} = \omega \qquad x = jz \Rightarrow z$ real

$\frac{e^{z} + e^{-z}}{2} = \omega \qquad x = jz$

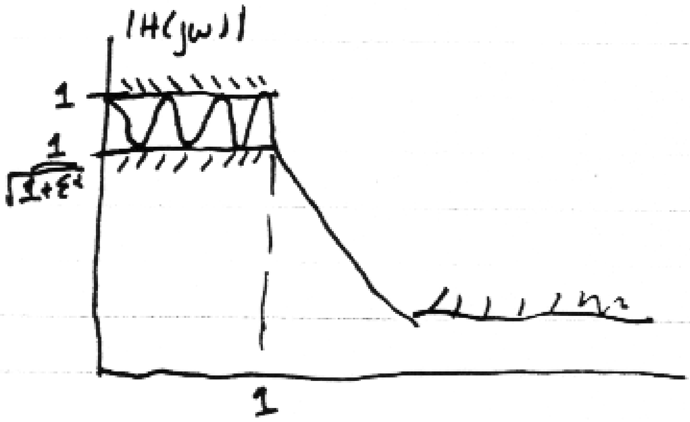
$\cos(x) = \omega$

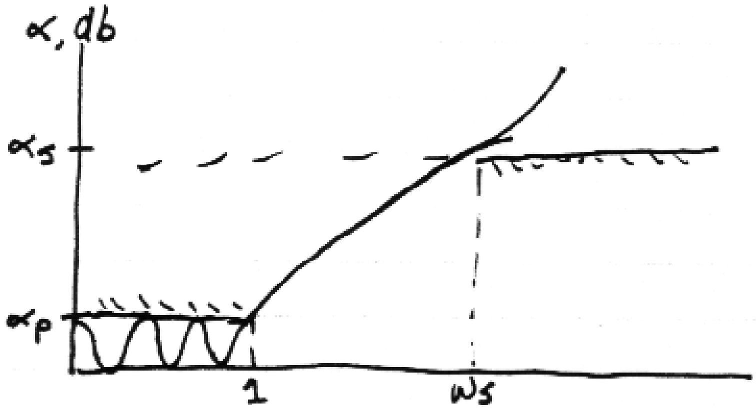
$C_N(\omega) = \cos(Nx) = \cos(Njz) = \cosh(Nz) \Rightarrow \boxed{\cosh(N\cosh^{-1}\omega}$

$\cos(jz) = \omega \Rightarrow \cosh(z) = \omega$

$z = \cosh^{-1}\omega$

# DESIGN EQUATIONS of normalized Chebyshev I





N=? ε=? poles? PASSIVE / ACTIVE.

$$\alpha(\omega) = -20\log_{10}\{1+\varepsilon^2 C_N^2(\omega)\}$$

$$\alpha(1) = \alpha_p = 10\log_{10}\{1+\varepsilon^2 C_N^2(1)\} = 10\log_{10}\{1+\varepsilon^2\}$$

$$10^{\frac{\alpha_p}{10}} = 1+\varepsilon^2 \Rightarrow \varepsilon = \sqrt{10^{\alpha p/10}-1} \quad (-1)$$

Finding order N

For $\omega > 1$ $C_N(\omega) = \cosh(N\cosh^{-1}\omega)$

$$\alpha(\omega_s) = \alpha_s = 10\log_{10}(1+\varepsilon^2 C_N^2(\omega_s))$$

$$10^{\frac{\alpha_s}{10}} = 1+\varepsilon^2 C_N^2(\omega_s)$$

$$10^{\frac{\alpha_s}{10}} = 1+\varepsilon^2(\cosh(N\cosh^{-1}\omega_s))^2$$

$$\frac{10^{\frac{\alpha_s}{10}}-1}{\varepsilon^2} = (\cosh(N\cosh^{-1}\omega_s))^2$$

$$\therefore \cosh(N\cosh^{-1}\omega_s) = \sqrt{\frac{10^{\frac{\alpha_s}{10}}-1}{10^{\frac{\alpha_p}{10}}-1}}$$

$$N\cosh^{-1}\omega_s = \left\{\sqrt{\frac{10^{\frac{\alpha_s}{10}}-1}{10^{\frac{\alpha_p}{10}}-1}}\right\}\cosh^{-1}$$

$$N = \text{ceil}\left[\frac{\cosh^{-1}\left(\sqrt{\frac{10^{\alpha s/10}-1}{10^{\alpha p/10}-1}}\right)}{\cosh^{-1}\omega s}\right]$$

→ Finding the poles (normalized case)

$$|H_{ch}(j\omega)|^2 = H_{ch}(j\omega)\,H_{ch}(-j\omega) = \underset{ch}{H}(s)\ \underset{ch}{H}(-s)\Big|_{s=j\omega}$$

$$H(s)H(-s) = |H(j\omega)|\Big|_{\omega=\frac{s}{j}} = \frac{1}{1+\varepsilon^2\cos^2(N\cos^{-1}s(j))}$$

Poles are equal to the roots of

$$1+\varepsilon^2\cos^2(N\cos^{-1}s(j))\cancel{3} = \emptyset$$

$$\cos^{-1}\left(\frac{s}{j}\right) = \underset{\text{complex}}{\omega} = u+jv \Rightarrow \frac{s}{j} = \cos(\omega) = \cos(u+jv)$$

$$s = j\left\{\cos(u+jv)\right\} = j\left\{\cos u\cos jv - \sin u\sin jv\right\}$$

$$= j\{\cos u \cosh v - j \sin u \sinh v\}$$

$$= \sin u \sinh v + j \cos u \cosh v = \underset{\text{real}}{\overset{}{\underset{\uparrow}{\sigma}}} + j\omega \leftarrow \text{imag.}$$

$$\Rightarrow \quad 1+\varepsilon^2\cos^2(N\omega) = 0$$

$$\cos^2(N\omega) = \frac{-1}{\varepsilon^2} \Rightarrow \cos(N\omega) = \pm\frac{j}{\varepsilon}$$

$$\cos(Nu + jNv) = \pm\frac{j}{\varepsilon}$$

$$\cos Nu \cos jNv - \sin Nu \sin jNv = \pm\frac{j}{\varepsilon}$$

$$\cos(Nu)\cosh(Nv) - j\sin(Nu)\sinh(Nv) = \pm\frac{j}{\varepsilon}$$

$$\cos(Nu)\cosh(Nv) = 0 \Rightarrow \cos(Nu) = 0$$

$$Nu = (2k+1)\frac{\pi}{2}, \quad k = 0, 1, 2, 3, \ldots$$

$$u_k = \frac{(2k+1)\pi}{2N}, \quad k = 0, 1, 2, 3, \ldots$$

$$\sin(Nu)\sinh(Nv) = \pm 1/\varepsilon \Rightarrow \sinh(Nv) = \pm 1/\varepsilon$$

$$Nv = \pm \sinh^{-1} 1/\varepsilon$$

$$V_k = \pm\frac{1}{N}\sinh^{-1}\left(\frac{1}{\varepsilon}\right) \quad k = 0, 1, 2, 3, \ldots$$

$$S = \sin u \sinh v + j \cos u \cosh v$$
$$= \sigma + j\omega$$

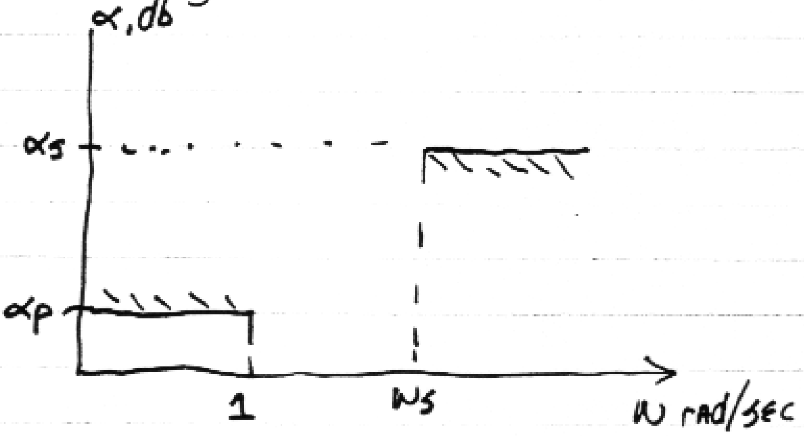
Real part $\rightarrow \sigma_k = - / \sinh\left(\frac{1}{N} \sinh^{-1} \frac{1}{\epsilon}\right) \sin \frac{(2k+1)\pi}{2N}$

$\omega_k = \cosh\left(\frac{1}{N} \sinh^{-1} \frac{1}{\epsilon}\right) \cos\left(\frac{(2k+1)\pi}{2N}\right)$

$\Big\} k = 0, 1, 2, 3,$

TABLES page 248

# DESIGN Chebyshev I LPF



$$\alpha(\omega) = 10\log_{10}\{1+\varepsilon^2 C_N^2(\omega)\}$$

$$\varepsilon = \sqrt{10^{\alpha p/10}-1} \quad ①$$

$$N = \text{CIEl}\left(\frac{\sqrt{\frac{10^{\alpha s/10}-1}{10^{\alpha p/10}-1}}}{\cosh^{-1}\omega s/\omega p}\right) \quad ②$$

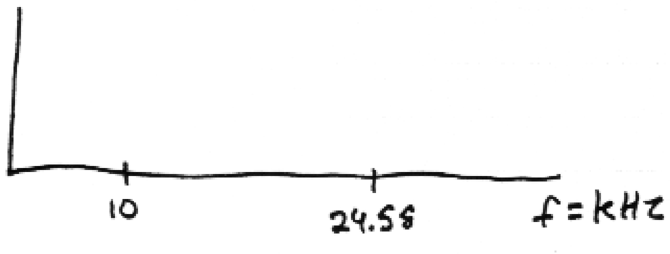
Poles: $P_K = \sigma_K + j\omega_K$

where $\sigma_K = -\left|\sinh\left\{\frac{1}{N}\sinh^{-1}\frac{1}{\varepsilon}\right\}\frac{\sin(2k+1)\pi}{2N}\right|$

& $\omega_K = \cosh\left\{\frac{1}{N}\sin^{-1}\frac{1}{\varepsilon}\right\}\frac{\cos(2k+1)\pi}{2N}$

$k = 1, 2, 3, \ldots, N$ ← order

Example | Design a LPF with $f_p = 10$KHz w/ $\alpha_p = .3$db & $f_s = 24.58$KHz & $\alpha_s = 22$db



$\omega_p = 2\pi \times 10^4$ rad/sec

$\omega_s = 2\pi \times 24.58 \times 10^3$ rad/sec

Normalized so $\omega_p = 1 rad/sec$

$$\boxed{\therefore K_f = 2\pi \times 10^4}$$

Find the order $N = 3$

$$\varepsilon = \sqrt{10^{.3/10} - 1} =$$

Find the poles:

$$\sigma_K = -\left| \sinh \left\{ \frac{1}{N} \sinh^{-1} \frac{1}{\varepsilon} \right\} \sin \frac{(2k+1)}{2N} \pi \right|$$

$$\omega_K = \cosh \left\{ \frac{1}{N} \sinh^{-1} \frac{1}{\varepsilon} \right\} \frac{\cos (2k+1)\pi}{2N}$$

N odd ⇒ Real pole: $-\left| \sinh \frac{1}{N} \sinh^{-1} \frac{1}{\varepsilon} \right| = -0.7293$ rad/sec
(k=1)

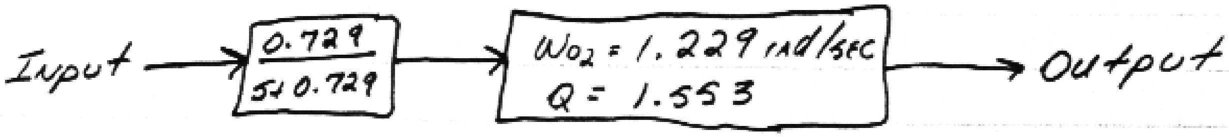
Complex Poles: $P_2, P_3 = -0.3646 \pm j\, 1.072$

$$\omega_{02} = \sqrt{(0.3646)^2 + (1.072)^2} = 1.22906 \text{ rad/sec}$$

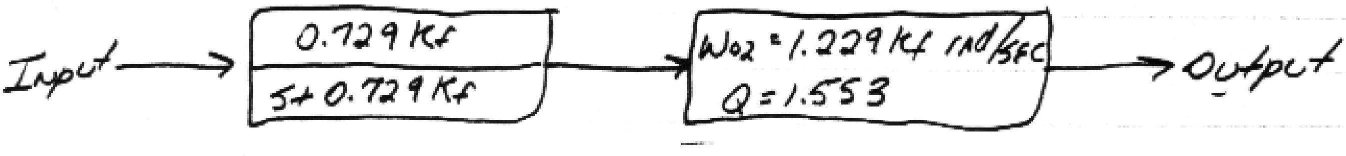
$$Q_2 = \frac{\omega_0}{2 \times 0.3646} = 1.553$$

Realization

Normalized filter



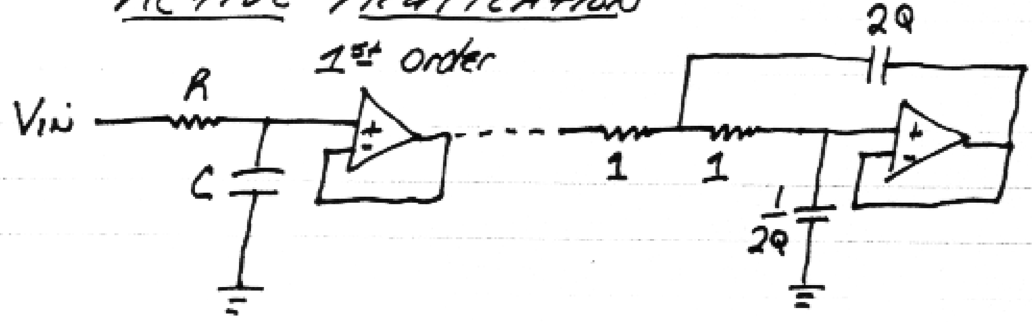
Denormalized filter: $K_f = 2\pi \times 10^4$



PAGE 248 Chebyshev pole locations

$P = -\alpha + j\beta$

<u>ACTIVE REALIZATION</u>



$K_f = 2\pi \times 10^4 \cdot 1.229$

$K_m = ?$

$RC = \frac{1}{0.729 K_f}$

ASSUME C to find R

$\omega_{o_2} = K_f \times 1.229 = 1.229 \cdot 2\pi \times 10^4$

$Q_2 = 1.553$

DESIGN OF PASSIVE Chebyshev I LPF

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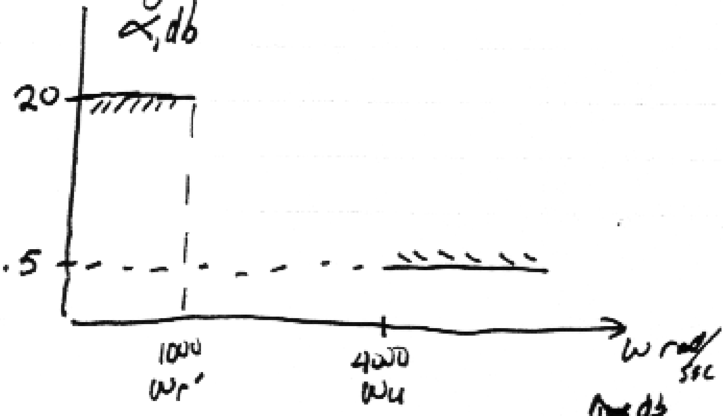
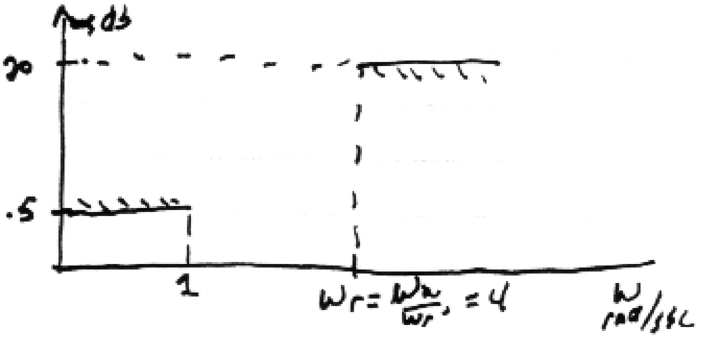
→ <u>HPF DESIGN</u>

<u>Example</u>:

DESIGN A HPF with Ch, approx. to meet the following specs.



HPF → LPF x-form



$N = 2$ (from x-fûn)

$\varepsilon = \sqrt{10^{.05} - 1} = 0.3493$

From tAblE: PolEs: $-.7128 \pm j1.004$

$\omega_{o(LP)} = \sqrt{(.7128)^2 + (1.004)^2} =$

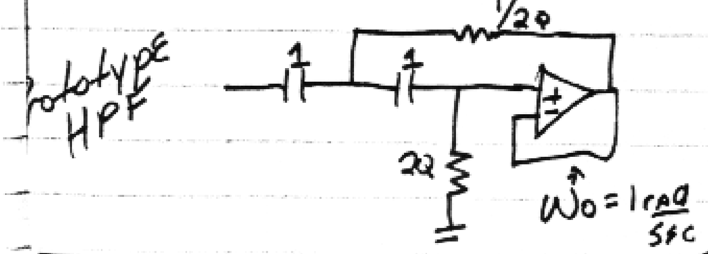
$Q = \dfrac{\omega_{o(LP)}}{2 \cdot 0.7128} = .863$

$$G_{LP}(s) = \frac{\omega_o(LP)}{s^2 + s\frac{\omega_{o(LP)}}{Q} + \omega_o^2(LP)}$$

$$H_{HP}(s) = G(s)\Big|_{s=\frac{\omega_u}{s}} = \frac{s^2}{s^2 + s\frac{\omega_{o(HP)}}{Q_{HP}} + \omega_{o(HP)}^2}$$

$Q_{HP} = Q_{LP} = .863$

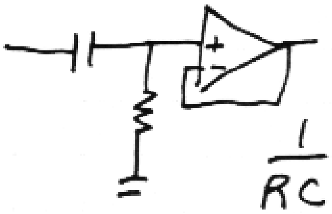
$\omega_{oHP} = \dfrac{\omega_u}{\omega_{o(LP)}} = 3252 \ \frac{rad}{sec}$



$K_f = 3252 \ \frac{rad}{sec}$

$Q = .863$

---

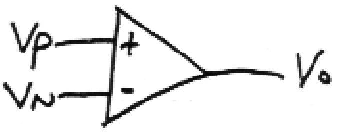


$\dfrac{1}{RC} = \dfrac{\omega_u}{\sigma_p}$

$$\begin{tikzcd}
\frac{s}{s+\frac{\omega_u}{\sigma_p}} & \frac{\sigma_p}{s+\sigma_p} \arrow[l, "LP"', "HP"] \arrow[d] \\
& \frac{\sigma_p}{\frac{\omega_u}{s}+\sigma_p} \arrow[ul, bend left]
\end{tikzcd}$$

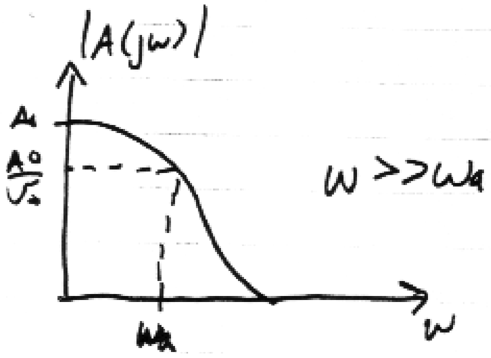
First (Real)
Order
Pole

## The real opamp model 6/22/98



$V_o = A(s)(V_p - V_N)$

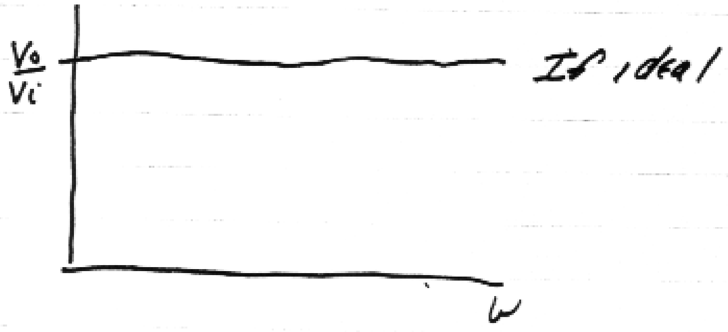
$A(s) = \frac{A_0}{1+s/\omega_0}$  $A(j\omega) = \frac{A_0}{1+j\omega/\omega_0}$



$\omega >> \omega_a$

$A(j\omega) = \frac{A_0 \omega_a}{j\omega}$  $A_s = \frac{GB}{s}$ ⇒ gain bandwidth product

## Non-inverting amp



Analysis: $\frac{R}{R+(k-1)R} V_o = V_N = \frac{V_o}{k}$ (-1)

$V_o = A(s) V_p - V_N = A(s)(V_i - V_N)$ (-2)

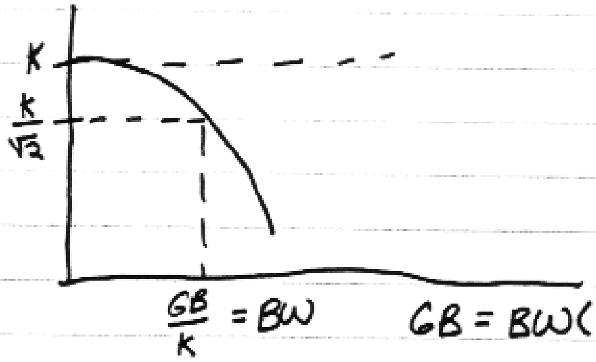
$V_o = A(s)(V_i - \frac{V_o}{k})$  $\frac{V_o}{A(s)} + \frac{V_o}{k} = V_i$

$V_o\left(\frac{1}{A(s)} + \frac{1}{k}\right) = V_i \quad \Rightarrow \quad \frac{V_o}{V_i} = \frac{K}{1 + K/A(s)}$

$\frac{V_o}{V_i} = \frac{K}{1 + sK/GB} \Leftarrow$ 1st order LPF

Freq. Response: $H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{K}{1 + j\omega K/GB}$

$|H(j\omega)| = \frac{K}{\sqrt{1 + \left(\frac{\omega k}{GB}\right)^2}}$



$GB = BW(K)$

$\measuredangle H(j\omega) = -\tan^{-1}\frac{\omega k}{GB}$

<u>Error</u> <u>Analysis</u>

① Magnitude Response Error

$|H(j\omega)| = \frac{K}{\sqrt{1 + \left(\frac{\omega k}{GB}\right)^2}}$ Assume $\omega << \frac{GB}{K}$

$|H(j\omega)| \cong K\left\{1 - \frac{1}{2}\left(\frac{\omega k}{GB}\right)^2\right\}$ % Mag. Error $\cong -\frac{1}{2}\left(\frac{\omega k}{GB}\right)^2$

very small $\rightarrow$

## ② Phase Response Error

$$\theta(j\omega) = \measuredangle H(j\omega) = -\tan^{-1}\left(\frac{\omega k}{GB}\right)$$

$$\tan^{-1} = x - \frac{x^3}{3} + \ldots\ldots \qquad |x| << 1$$

$$\tan^{-1} x \cong x$$

$$\theta(j\omega) = \frac{-\omega k}{GB}$$ ⇐ Phase Error in Radians

Example/

→ $k = 3$ → opamp w/ $GB = 2\pi \times 1.5 \times 10^6$ rad/sec

→ freq. range 0-50 kHz

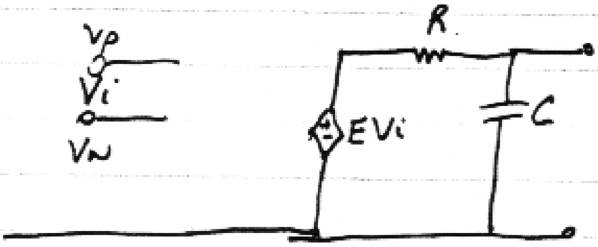
→ Find the phase error in degrees. ←

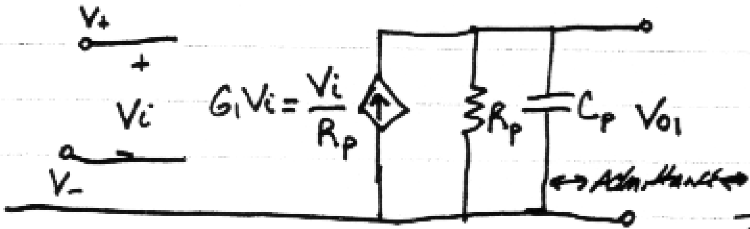
$$\theta(\omega) = \frac{-2\pi\ 50 \times 10^3\ (3)}{2\pi \times 1.5 \times 10^6} \text{ (radians)}$$

$$\theta(\omega) \text{ in degrees} = \frac{-2\pi\ 50 \times 10^3 (3)}{2\pi \times 1.5 \times 10^6} \cdot \frac{180}{\pi} = -5.7^\circ$$

# Simple Opamp Macro Model

$$A(s) = \frac{A_o}{1+ s/\omega_a} = \frac{V_o}{V_i}$$



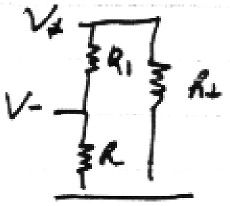


$$\frac{V_{o1}}{V_i}$$

$$\frac{V_{o1}}{V_i} = \frac{G_1 \cancel{V_i}}{\frac{1}{R_p} + sC_p} = \frac{G_1 R_p}{1+ sC_pR_p}$$

$$\frac{G_1 R_p}{1+ sC_pR_p} = \frac{A_o}{1+\frac{s}{\omega_a}} \qquad A_o = G_1 R_p \quad ①$$

$$\omega_a = \frac{1}{C_pR_p} \quad ②$$



$R_1 = 1M\Omega$

$R_+ = 10M\Omega$

$R_- = 10M\Omega$

For GB = 1MHz

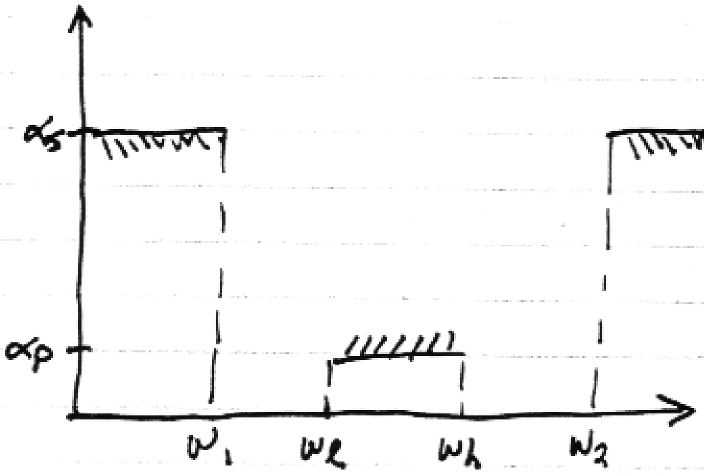
$G_1 = 0.6283$ $\qquad R_o = 75\Omega$

$G_2 = 1/75 = .01333$ $\qquad R_p = 159.155k\Omega$

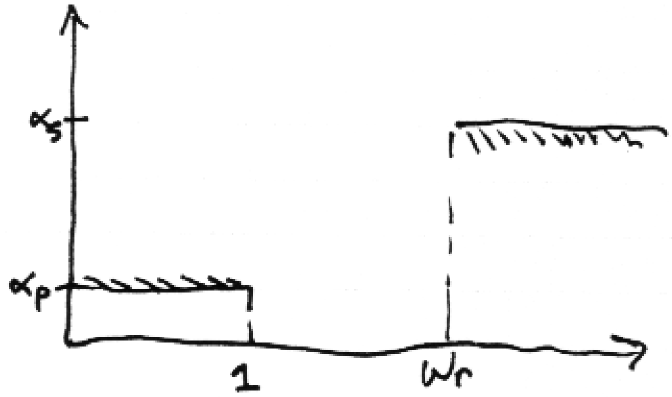
$C_p = 0.1\,uF$

# DESIGN ACTIVE BPF

6/24/98



BP → LP X-form



$$H_{LP}(s) = \frac{\overset{\swarrow 1^{st}\text{ order}}{\varepsilon}}{s+\varepsilon} \cdot \frac{\omega_{o1}^{2}(LP)}{s^2 + \frac{s\omega_{o1}(LP)}{Q_1} + \omega_{o1}^{2}(LP)} \cdot \cdot \cdot \cdot \cdot$$

$$H_{BPF}(s) = H_{LP}(s)\Big|_{s = \frac{s^2+\omega_0^2}{s\Delta\omega}} \qquad Q = \frac{q_c}{\varepsilon} \qquad q_c = \frac{\omega_0}{\Delta\omega}$$

# Geffes' Algorithm

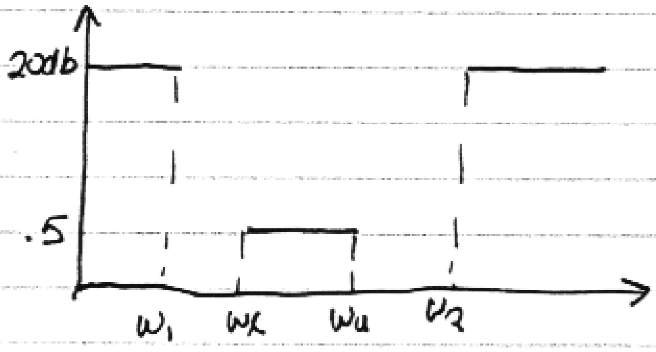
$C = \Omega_2^{\,2} + E^2$ $\qquad G = \sqrt{E^2 - 4D^2}$

$D = \dfrac{2E_0}{q_c}$ $\qquad Q_{BP} = \dfrac{1}{D}\sqrt{\dfrac{E+G}{2}}$

$E = 4 + \dfrac{C}{q_c^{\,2}}$ $\qquad K = \dfrac{E_2 Q_{BP}}{q_c}$

$W = K + \sqrt{K^2 - 1}$ $\qquad \omega_{o_1(BP)} = \dfrac{\omega_o}{W}$ $\qquad \omega_{o_2(BP)} = \omega_o W$

## Example

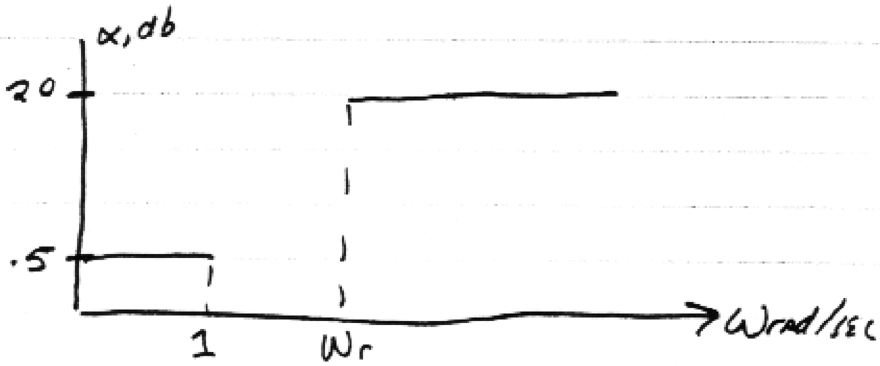


$W_l = 500$ rad/sec
$W_u = 1000$ " "
$W_1 = 333$ " "
$W_2 = 1500$ " "

$W_o = \sqrt{w_l w_u} = 707.1$ rad/sec $\qquad q_c = W_o/BW = 1.414$

$BW = W_u - W_l = 500$ rad/sec

$A = 2.336$
$B = 2.333$
$W_r = 2.333$



Choose Chebyshev I Approx.

N = 3 → Find poles from the table. ←

$P_1 = -0.6265$ (Real)

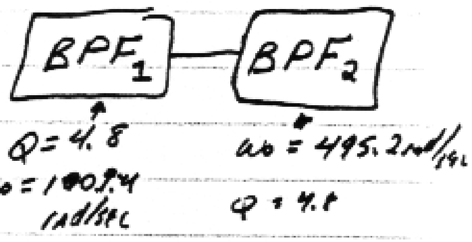
$$T_1(s) = \frac{\varepsilon}{s+\varepsilon} = \frac{.6265}{s+.6265}$$

$$P_{2,3} = -0.3132 \pm j1.022$$

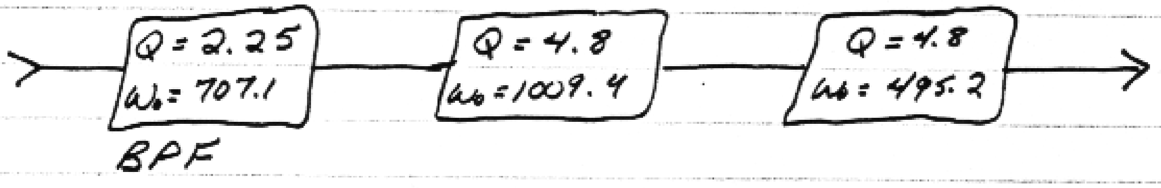
$$= -\varepsilon_2 \pm j\Omega_2$$

$T_2(s) =$

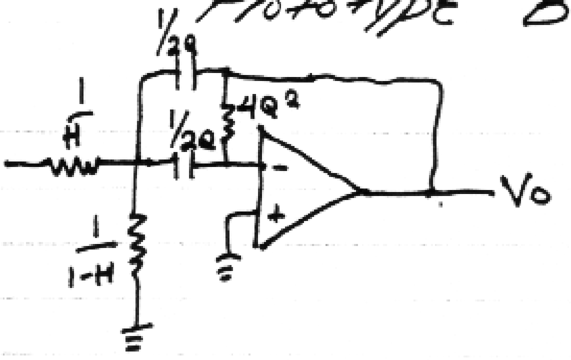
Consider $T_2(s)\Big|_{s=\frac{s^2+\omega_0^2}{sBW}}$ ⇒



Final Filter



## Prototype BPF Circuit



$\omega_0 = 1$ rad/sec

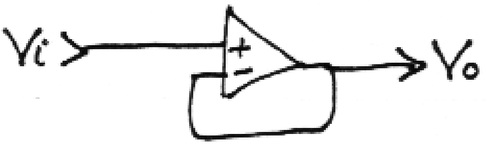
$$\frac{V_o(s)}{V_i(s)} = \frac{-(Hs/Q)2Q^2}{s^2+\frac{s}{Q}+1}$$

$H<1$

1st stage $Q = 2.757$ $\omega_0 = 707.1$ $H_1 = ?$

$K_{f_1} = 707.1$ $K_m = ?$

## The slew rate effect



$V_o = V_i$ (ideal opamp)

MAX volt change / time ⇒ slew rate

741 1V/us Better 1000V/us

MAX happens where sin( ) crosses X-axis

$$\left.\frac{dv}{dt}\right|_{max} = V_x \omega \cos\omega t \Big|_{t=0} = \boxed{V_x\omega \le SR}$$