ECE 480

Spring/Summer 2024

Prerequisite Assessment Test

"Write out and sign the Honor Pledge"

(1) Find the result of the following complex expressions as magnitude and phase (degrees) as well as real and imaginary components:

$$A = 2 - j2,\ B = 2 - j2\ C = 2 - j3\ , D = -5 - j4, E = 2 + j,\ F = 4j^{-j},$$

and

$$G = A^3B^2C^3/(D^2E^4F).$$

(2) Consider a LTI system with the following transfer function

$$H_a(s) = N_a(s)/D_a(s),$$

where

$$D_a(s) = s^2 + (\Omega_0/Q)s + \Omega_0^2,$$

$$N_a(s) = k\left(s^2 - (\Omega_0/Q)s + \Omega_0^2\right),$$

and given the following information:

- The zeros are given by: $1.256637 \times 10^5\ \angle(\pm 95.73917^\circ)$.
- The poles are the complex conjugate of the zeros.
- The DC magnitude response is equivalent to 21.5836 dB.

(a) From the above information, find the following parameters: $Q,\ \omega_0,\ and\ k.$

(b) Derive expressions for the magnitude and phase responses. Hence, plot it in order to determine the type of filter. What is it?

(c) Plot the magnitude response in dB, and the phase response in degrees versus the frequency in Hz.

(d) Find the frequencies at which the phase responses are:

- -47.2232 deg.

- -90 deg.

(e) Plot the group delay in msec.

(3) Find the first 6 terms of the Taylor's series expansion of the following functions:

(a) $y = \dfrac{1}{(1-0.5x^2)^{3/2}}$.

(b) $y = \tanh(x^{-1})$, $x \neq 0$. Hint: Write $y = \sum_{i=0}^{\infty} a_i x^{-i}$

(c) $y = 2.302585 \log_{10}(7.389056 - 3.694528x)$, where $|x| < 2$

(d) $y = a^x$, where $a = 20.0855$.

(4) Find the following:

(a) $\sum_{n=0}^{\infty} (-1.25)^{-2n}$

(b) $\sum_{n=-4}^{5} (0.75)^n$

(c) $\sum_{n=0}^{6} 0.5^{-n}$

(d) $\sum_{n=0}^{8} n(n-4)$

(5) Expand the following signals in terms of their sinusoidal components. Hence, plot their line spectra and calculate their average power:

(a) $\sin\left(12\pi t + \frac{\pi}{3}\right)\cos(8\pi t)$

(b) $\cos^2\left(6\pi t + \frac{\pi}{3}\right)$

(c) $\cos^5(12\pi t)$

(6) Show that:

$$2\cos(2\pi f_1 t + \pi/3) + 4\cos(2\pi f_1 t + \pi/4) + 5cos(2\pi f_1 t - \pi/6),$$

can be reduced to one sinusoid such as K $\cos(2\pi f_1 t + \phi)$, where $f_1 = 1\ kHz$. Hence, find K and ϕ.

(7) Solve the 2 simultaneous system of equations to obtain the real roots:

$$\begin{aligned} y - 50\ tan^{-1}(0.25x) &= 0 \\ y - (4 - 0.5x)^2 &= 0. \end{aligned}$$

(8) Consider the following signal $x_a(t) = 5 + cos^3(40\pi t) + 2\cos(60\pi t + \pi)\sin(140\pi t)$

(a) Write $x_a(t)$ as the sum of 4 sinusoids, in addition to a dc component. Hence, plot it.

(b) Plot the line spectrum of the signal $x_a(t)$.

(c) Determine the average power of $x_a(t)$.

(d) Consider the first order filter with transfer function given by

$$H_a(s) = \frac{s - \omega_0}{s + \omega_0},$$

where $\omega_0 = 200\pi$ rad./sec. Determine its magnitude and phase frequency responses. Hence, write the steady-state output response of the filter if the input signal is $x_a(t)$.

(e) Determine the average power of the steady-state output.

(f) Derive and plot the step-response of the above filter

Verify your answers using Matlab whenever possible.