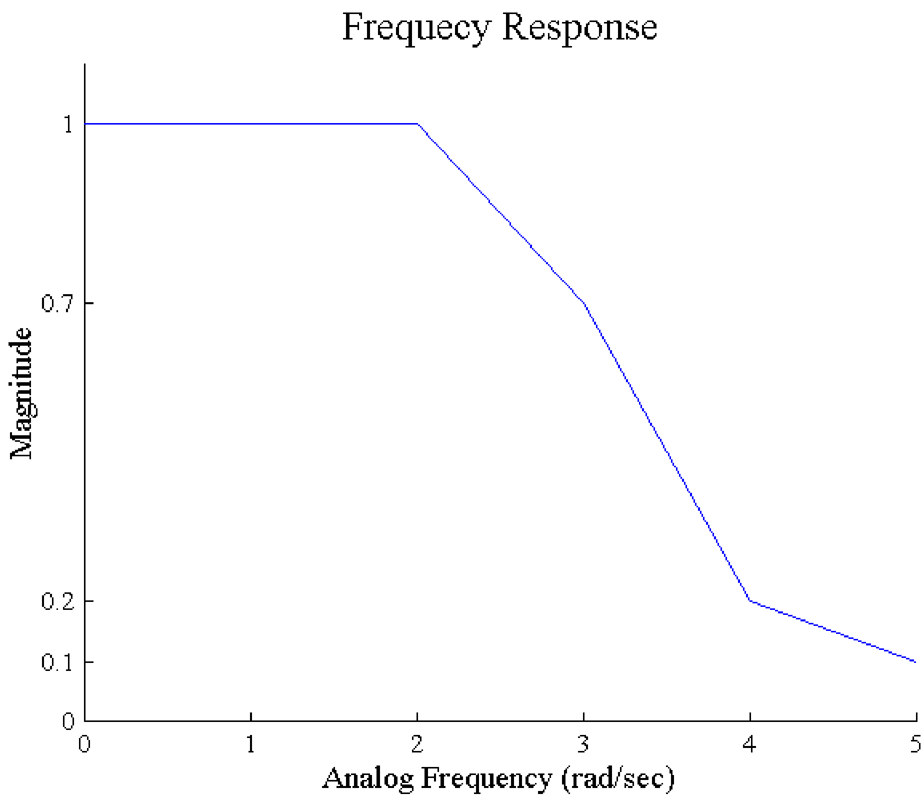
HW #5

1) An analog filter specified by an H(s) has the frequency response shown below. A digital filter H(z) is formed by replacing s by $\frac{4(1-z^{-1})}{(1+z^{-1})}$.

Roughly draw the 20 log magnitude (magnitude in dB) of the frequency response of the filter H(z).



Bilinear transformation equation: $$H_{(Z)} = H_{a(S)}\Big|_{S=\frac{2\left(1-z^{-1}\right)}{T\left(1+z^{-1}\right)}}$$

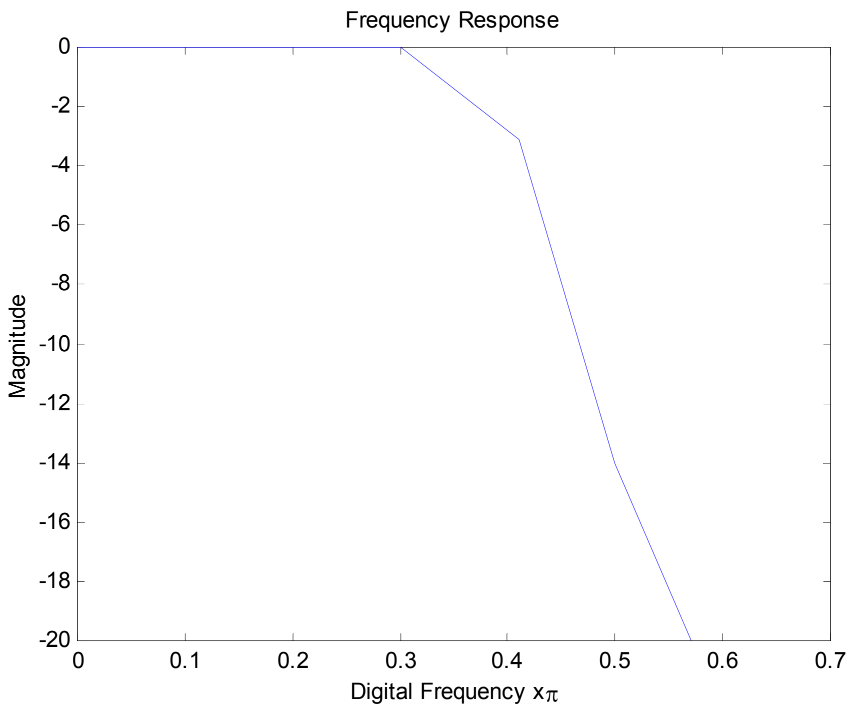
if $s = \dfrac{4\left(1-z^{-1}\right)}{\left(1+z^{-1}\right)}$ → then $4 = \dfrac{2}{T}$ → $\therefore T = \dfrac{1}{2}$

$\Omega =$ ANALOG FREQUENCY $\qquad \Omega = \dfrac{2}{T}\mathrm{TAN}\left(\dfrac{\omega}{2}\right)$

ω = DIGITAL FREQUENCY $\qquad \omega = 2\mathrm{TAN}^{-1}\left(\dfrac{\Omega T}{2}\right)$

Calculate the first 5 points using the equations above and plot out the 20 log magnitude of the frequency response.

Ω	0	1	2	3	4	5
ω	0	0.15π	0.3π	0.41π	0.5π	0.57π
$H(e^{j\omega})$	1	1	1	0.7	0.2	0.1
$20\log\lvert H(e^{j\omega})\rvert$	0	0	0	-3.1	-14	-20



2) Using the bilinear transformation $s = \frac{1-z^{-1}}{1+z^{-1}}$, what is the image of $s = e^{j\frac{\pi}{2}}$ in the z-plane?

Bilinear transformation equation: $$H(z) = H_a(s)\,\Big|_{S=\frac{2\left(1-z^{-1}\right)}{T\left(1+z^{-1}\right)}}$$

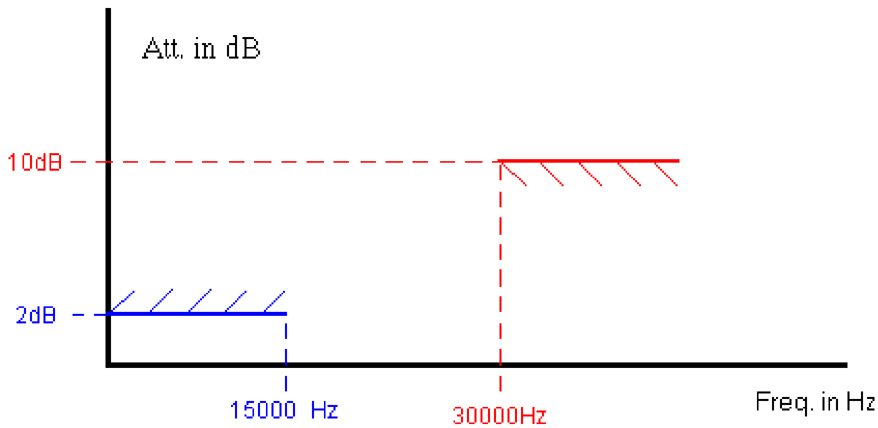
$$s = \frac{\left(1-z^{-1}\right)}{\left(1+z^{-1}\right)} \quad \text{or} \quad z = \frac{(1+s)}{(1-s)}, \textit{where}\, T = 2$$

if $s = e^{j\frac{\pi}{2}} = j$ then $z = \frac{1+j}{1-j} = \frac{(1+j)^2}{(1-j)(1+j)} = \frac{(1+j)^2}{2} = j$

This is a coincidence, i.e. s = z.

3) Design a digital filter H(z), using the bilinear transformation method to be used in an A/D-H(z)-D/A structure to satisfy the following analog specifications (use Chebyshev prototype).

a) Sampling rate of 60,000 samples/sec
b) Low-Pass filter with –2dB cutoff at 15,000Hz
c) Stop-Band attenuation of 10dB at 30,000Hz and greater



$f_p = 15$ kHz, $f_s = 30$ kHz, $f_{samp} = 60$ kHz

$$T = \frac{1}{f_{samp}} = \frac{1}{60 \cdot 10^3}$$

The equivalent digital specifications will be:

$\omega_p = 2\pi f_p T = 0.5\pi$ rad/sec
$\omega_s = 2\pi f_s T = \pi$ rad/sec

Next prewarp the above frequencies to prepare for the bilinear transformation:

Take T = 1 sec

$$\Omega'_p = \frac{2}{T}\mathrm{TAN}\left(\frac{\omega_p}{2}\right) = 2.0\mathrm{rad/sec}$$

$$\Omega'_s = \frac{2}{T}\mathrm{TAN}\left(\frac{\omega_s}{2}\right) = \infty$$

Assuming a Chebyshev approximation (Ch I) and n =1 (first order system).

$$H_a(s) = \frac{2.6151}{s + 2.6151}$$

Applying the bilinear transform:

$$H(z) = H_a(s)\Big|_{s=\frac{1-z^{-1}}{1+z^{-1}}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.567(1+z^{-1})}{1+0.133z^{-1}}$$

$$\therefore y(n) + 0.133y(n-1) = 0.567x(n) + 0.567x(n-1)$$

4) Design a digital high-pass filter H(z), using the bilinear transformation method to be used in an A/D-H(z)-D/A structure to satisfy the following analog specifications, and a sampling rate of 1,000 samples/sec.

$\Omega_s = 199.34 \text{ rad/sec}$ $\quad\quad\quad$ $\Omega_p = 927.3 \text{rad/sec}$

$\alpha_s = 20 \text{ dB}$ $\quad\quad\quad$ $\alpha_p = 3 \text{ dB}$

a) Find H(z) and plot –20 log |H($e^{j\omega}$)| and arg H($e^{j\omega}$).

Analog to digital conversion:

$$\omega_s = \Omega_s * T_s = \frac{199.34}{1000} = 0.19934 \text{rad/sec} \qquad \omega_p = \Omega_p * T_s = \frac{927.3}{1000} = 0.9273 \text{rad/sec}$$

Designing the digital filter: Assume T = 1, and prewarp the frequencies to obtain the analog requirements.

$$\Omega'_p = \frac{2}{T} \text{TAN}\left(\frac{\omega_p}{2}\right) = 1 \text{rad/sec}$$

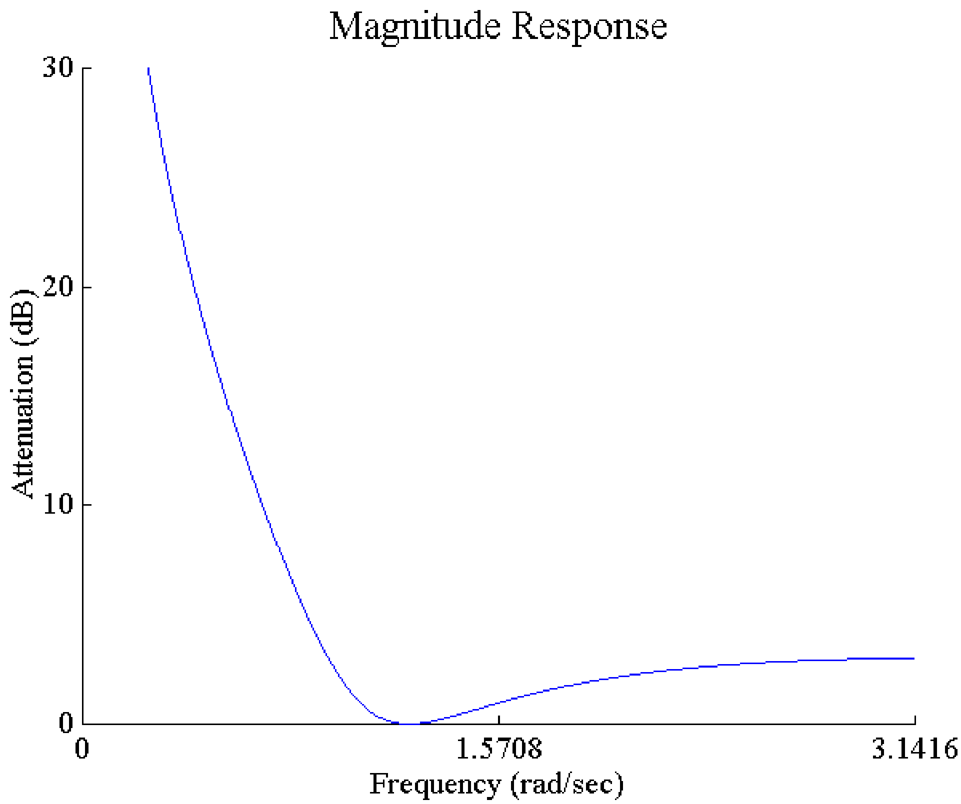
$$\Omega'_s = \frac{2}{T} \text{TAN}\left(\frac{\omega_s}{2}\right) = 0.2 \text{rad/sec}$$

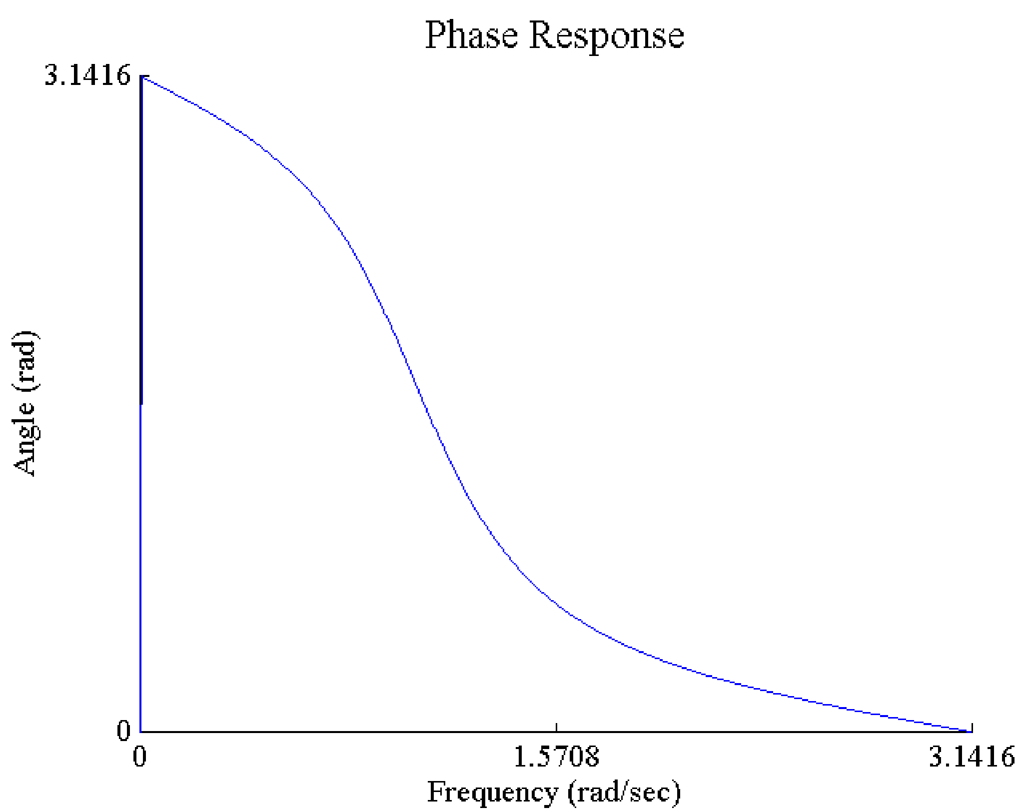
The filter designed above requires a 2nd order Chebyshev I filter.
n=2

$$H_a(s) = \frac{0.501189 s^2}{0.7079 s^2 + 0.645 s + 1} \text{ can also be written as } H_a(s) = \frac{0.7079 s^2}{s^2 + 0.9109 s + 1.4126}$$

Apply the bilinear equation:

$$H(z) = H_a(s)\Big|_{s=\frac{1-z^{-1}}{1+z^{-1}}} = \frac{0.3914\left(1 - 2z^{-1} + z^{-2}\right)}{1 - 0.7153 z^{-1} + 0.496 z^{-2}}$$


Magnitude Response
30
20
10
0
Attenuation (dB)
0
1.5708
3.1416
Frequency (rad/sec)


Phase Response
3.1416
0
Angle (rad)
0
1.5708
3.1416
Frequency (rad/sec)

c) What limitations must be placed on the input signals so that the A/D-H(z)-D/A structure truly acts like a high-pass filter?

The signal must be smaller than the sampling frequency. In most A/D-H(z)-D/A structures filters it is wise to keep the maximum input frequency less than half of the sampling frequency.

5)

a) Design a low-pass digital filter that will operate on sampled data such that the analog cutoff frequency is 200 Hz (1 dB acceptable ripple) and at 400 Hz the attenuation is at least 20 dB with monotonic shape past 400 Hz. The sample rate is 2000 samples/sec.

$$\alpha_p = 1\text{dB} \qquad f_p = 200\text{Hz}$$
$$\alpha_s = 20\text{dB} \qquad f_s = 400\text{Hz}$$
$$T_s = \frac{1}{2000} = 0.5\text{msec}$$

Digital filter specifications:

$$\omega_p = \Omega_p * T_s = \frac{2*\pi*200}{2000} = 0.2\pi \ \text{rad/sec}$$
$$\omega_s = \Omega_s * T_s = \frac{2*\pi*400}{2000} = 0.4\pi \ \text{rad/sec}$$

Prewarping (T=1):

$$\Omega_p^{'} = \frac{2}{T}\text{TAN}\left(\frac{\omega_p}{2}\right) = 0.6498 \ \text{rad/sec}$$
$$\Omega_s^{'} = \frac{2}{T}\text{TAN}\left(\frac{\omega_s}{2}\right) = 1.4531 \ \text{rad/sec}$$

Design of analog filter:

Use Matlab:

```
» [N,Wn]=cheb1ord(0.6498,1.4531,1,20,'s')

N =

     3


Wn =

    0.6498
```

```
» [B,A]=cheby1(N,1,Wn,'s');
» Has=tf(B,A)

Transfer function:
           0.1348
----------------------------------
s^3 + 0.6423 s^2 + 0.523 s + 0.1348
```

Now apply the Bilinear transformation:

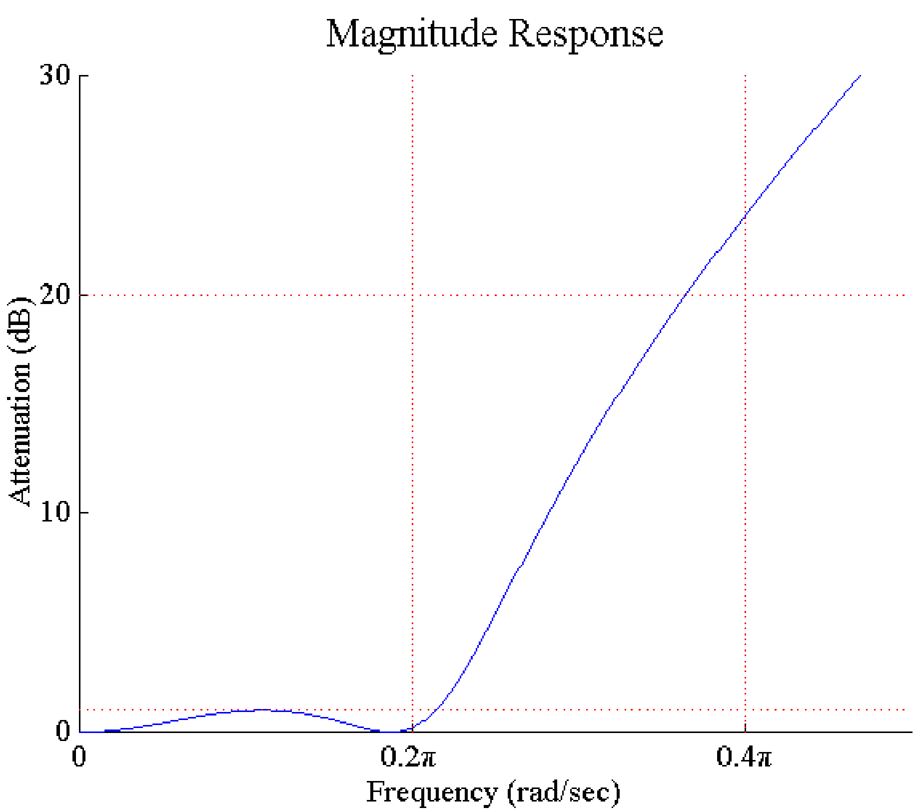
If T=1 ➔ f_s=1

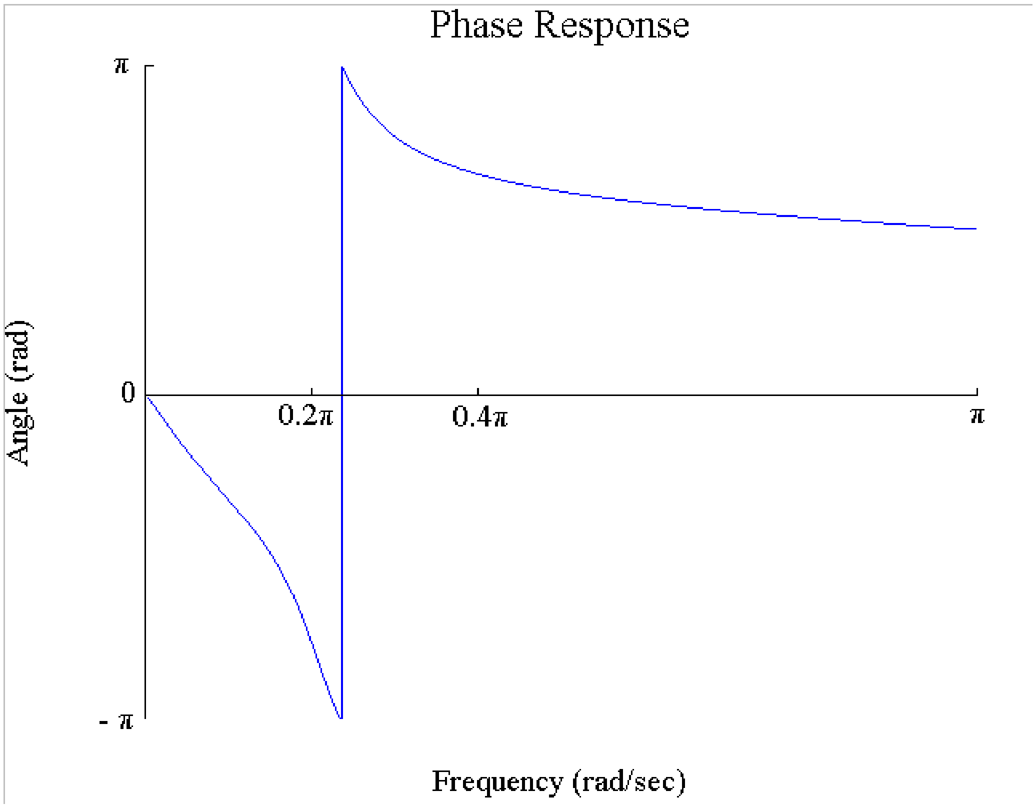
```
%% [Bdigital,Adigital]=bilinear(Banalog,Aanalog,fs);

» [Bdigital,Adigital]=bilinear(B,A,1);
» Hz=tf(Bdigital,Adigital,1)

Transfer function:
0.01147 z^3 + 0.03442 z^2 + 0.03442 z + 0.01147
---------------------------------------------
     z^3 - 2.138 z^2 + 1.769 z - 0.5398
```

b) For the designed filter in part (a), plot the -20 log |H($e^{j\omega}$)| and arg H($e^{j\omega}$)





c) Give the difference equation realization for your filter.

Matlab gives us:

Bdigital =

0.01147 0.03442 0.03442 0.01147

» Adigital

Adigital =

1.0000 -2.138 1.769 -0.5398

Therefore:

$$y(n) - 2.138\, y(n-1) + 1.769\, y(n-2) - 0.5398\, y(n-3) =$$
$$0.011147\, x(n) + 0.03442\, x(n-1) + 0.03442\, x(n-2) + 0.01147\, x(n-3)$$

e) How many real multiplications and additions are required for the implementation of this form?

We need 5 multiplications for this formula since some of the coefficients are repeated, and we need 6 additions for this formula.