A Priori & Post-Hoc Tests

Statistics
Hindsight is 20-20

- Although your data may suggest a new relationship, and thus new analyses...
- Theory should guide research and thus comparisons should be decided on before you conduct your experiment.
Planned & A Priori Comparisons

- Based on literature review
  - Theoretical

- Planned comparisons
  - A test that is conducted when there are multiple groups of scores, but specific comparisons have been specified prior to data collection.
  - A Priori Comparisons
Planned & A Priori Comparisons

If you have planned comparisons...

- Just run $t$ tests

- Subjective Decision about $p$ value
  - $p = .05?$
  - $p = .01?$
  - Bonferroni Correction?
Post-Hoc: Tukey HSD

- **Tukey Honestly Significant Difference**
  - Determines differences between means in terms of standard error
    - ‘Honest’ because we adjust for making multiple comparisons
    - The HSD is compared to a critical value

- **Overview**
  1. Calculate differences between a pair of means
  2. Divide this difference by the standard error
  * Basically this is a variant of a *t* test *
Tukey HSD

$$HSD = \frac{(M_1 - M_2)}{S_M}$$

$$t = \frac{(M_1 - M_2)}{S_{\text{Difference}}}$$

- For Tukey HSD, standard error is calculated differently depending on whether your sample sizes are equal.
Tukey HSD

- **Equal Sample Sizes**

\[ S_M = \sqrt{\frac{MS_{Within}}{N}} \]

\[ N = \text{Sample size within each group} \]

- **Unequal Sample Sizes**

\[ S_M = \sqrt{\frac{MS_{Within}}{N'}} \]

\[ N' = \frac{N_{Groups}}{\sum \left( \frac{1}{N} \right)} \]
Tukey HSD

- Determine Critical Value from Table
- Make a Decision
- Let’s do an example...
Tukey HSD: Example

- We will use the data from our One-Way ANOVA example:
  - Decision: Foreign graduate students in different programs place different importance on financial factors, on average.
  - Where are our differences?
### Importance Scores

<table>
<thead>
<tr>
<th>Source</th>
<th>4</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arts &amp; Sciences</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Education</td>
<td>4</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Law</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Business</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

### ANOVA Table

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>3.866</td>
<td>3</td>
<td>1.289</td>
<td>3.94</td>
</tr>
<tr>
<td>Within</td>
<td>4.256</td>
<td>13</td>
<td>0.327</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>8.122</td>
<td>16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Tukey HSD: Example

- Standard Error: Unequal Sample Sizes

\[
N' = \frac{N_{Groups}}{\sum \left( \frac{1}{N} \right)}
\]

\[
N' = \frac{4}{\frac{1}{5} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}} = \frac{4}{.95} = 4.211
\]

\[
S_M = \sqrt{\frac{MS_{Within}}{N'}}
\]

\[
S_M = \sqrt{\frac{.327}{4.211}} = .279
\]
Tukey HSD: Example

- Arts & Sciences (\(M=4.00\)) vs. Education (\(M=3.75\))

\[
HSD = \frac{(M_1 - M_2)}{s_M} = \frac{(4 - 3.75)}{.279} = .896
\]

- Arts & Sciences (\(M=4.00\)) vs. Business (\(M=3.75\))

\[
HSD = \frac{(M_1 - M_2)}{s_M} = \frac{(4 - 3.75)}{.279} = .896
\]

- Arts & Sciences (\(M=4.00\)) vs. Law (\(M=3.75\))

\[
HSD = \frac{(M_1 - M_2)}{s_M} = \frac{(4 - 2.75)}{.279} = 4.480
\]
Tukey HSD: Example

- **Critical Value**
  
  \( p = 0.05, \) 4 groups
  
  \[ \text{df}_{\text{within}} = 13 \]

- **A&S—Ed.**
  
  \( q = 0.896 \)

- **A&S—Bus.**
  
  \( q = 0.896 \)

- **A&S—Law**
  
  \( q = 4.480 \)

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**TABLE 10-13. EXCERPT FROM THE \( q \) TABLE**

Like the \( F \) table, we use the \( q \) table to determine critical values for a given \( p \) level, based on the number of means being compared and the within groups degrees of freedom. Note that critical values are in regular type for 0.05 and **boldface** for 0.01.

<table>
<thead>
<tr>
<th>WITHIN-GROUPS DEGREES OF FREEDOM</th>
<th>( k = ) NUMBER OF TREATMENTS (LEVELS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \cdot )</td>
<td>3</td>
</tr>
<tr>
<td>( \cdot )</td>
<td>4</td>
</tr>
<tr>
<td>( \cdot )</td>
<td>5</td>
</tr>
<tr>
<td>( 12 )</td>
<td>( 3.77 )</td>
</tr>
<tr>
<td></td>
<td>( 4.20 )</td>
</tr>
<tr>
<td></td>
<td>( 4.51 )</td>
</tr>
<tr>
<td>( 13 )</td>
<td>( 3.73 )</td>
</tr>
<tr>
<td></td>
<td>( 4.15 )</td>
</tr>
<tr>
<td></td>
<td>( 4.45 )</td>
</tr>
<tr>
<td>( 14 )</td>
<td>( 3.70 )</td>
</tr>
<tr>
<td></td>
<td>( 4.11 )</td>
</tr>
<tr>
<td></td>
<td>( 4.41 )</td>
</tr>
<tr>
<td>( \cdot )</td>
<td>( 4.89 )</td>
</tr>
<tr>
<td></td>
<td>( 5.32 )</td>
</tr>
<tr>
<td></td>
<td>( 5.63 )</td>
</tr>
</tbody>
</table>
Tukey HSD: Example

- Make a Decision
  - Foreign graduate students in Arts & Sciences place more importance on financial factors than students in Law but they do not differ from students in either Education or Business.