Interference and Diffraction of Light

Introduction
Reflection by mirrors and refraction by prisms and lenses can be analyzed using the simple ray model of geometric optics. However, when light passes through small apertures or holes, or scatters off surfaces having a closely-spaced periodic structure, the light spreads out, something like the spray from the nozzle of a garden hose. The phenomena that are observed under these circumstances cannot be explained using the simple geometric optics model. The wave nature of light must be explicitly taken into account in order to understand them.

Specifically, the phase relationship of two rays of light which come from the same source, but reach a point in space after having followed different paths in getting there must be taken into account. Basically, two extreme things can happen under these circumstances. If the two waves arrive at the point in phase, they add according to the principle of superposition (their electric field vectors are parallel) and the point is brighter than it would have been had either ray arrived by itself; a phenomenon referred to as constructive interference. If the two rays arrive out of phase, they cancel according to the principle of superposition (their electric field vectors are anti-parallel) and the point is less bright than it would have been, or may be entirely dark; a phenomenon referred to as destructive interference.

In this lab you will study three examples of such phenomena: transmission of light through a single narrow slit, transmission of light through two closely spaced narrow slits and transmission of light through multiple closely-spaced narrow slits

1. Single Slit Diffraction

Introduction
Consider light passing through a narrow aperture and falling on a viewing screen far from the aperture. The illumination at any point on the screen is the result of the combination of light from all the points within the aperture, and depends on the phase relation between the various waves from these points when they arrive at a single point of the viewing screen. This, in turn, depends on the distances from the various points in the aperture to the point on the screen. If all the waves arrive in phase, the light intensity at the point on the screen will be a maximum. Otherwise, not. The illumination on the screen consists of a pattern of light and dark regions called a diffraction pattern.

Consider a thin aperture or slit of width $a$ as shown in Figure 1. If such an aperture is illuminated by a coherent wave, the screen will show an intensity maximum in the straight ahead direction, $P_0$. For points on either side of $P_0$, the light from different locations within the aperture must follow paths of different lengths to get there, and phase differences develop. This results in a decrease in the intensity of light at these points.
Complete destructive interference (dark regions) occurs at points like \( P_1 \) which satisfy the equation,

\[
\sin \theta_m = \frac{m \lambda}{D}
\]

where \( \lambda \) is the wavelength of the light and \( m \) is an integer index taking on the values \( m = 1, 2, 3, ... \).

The first dark region occurs when \( m = 1 \), the second when \( m = 2 \), etc. Notice that \( \tan(\theta) = \frac{y}{D} \) in the figure. Thus, if we know \( a \) and measure \( y \) and \( D \) we can determine \( \theta \) through the relationship,

\[
\theta_m = \tan^{-1} \left( \frac{y}{D} \right)
\]

Using this angle and Equation 1 the wavelength of light, \( \lambda \), can be determined.

When light passes through a single slit such as has just been described, the illuminated pattern on the viewing screen generally looks like Figure 2 below.

Thus the width of the central maximum, a distance equal to \( 2y \), is the distance between the two \( m = 1 \) points of the complete destructive interference given by Equation 1.

**Procedure**

- At your lab station you should have an optics bench, a diode laser, a mounted disc containing sets of single slits of various lengths, sets of double slits and sets of multiple slits and a viewing screen with a millimeters scale attached to it. Place the diode laser at the 0 cm point on the optics bench. Place the slit apparatus at the 10 cm point and adjust it so that the laser beam illuminates the single slit having a stated width of 0.04 mm.
There was a manufacturing error in the production of this batch of slits. **Although the slit width is labeled 0.04 mm, this is not the actual slit width.** Your goal in this measurement is to determine experimentally the actual width of this slit.

- Place the viewing screen at the 110 cm point on the optics bench. You should see a pattern like *Figure 2* on the viewing screen.
- The wavelength of the light the diode produces is marked on the diode. Using this value and your measurements of $y$ and $D$, determine the width of the slit. Show your calculations.

\[
y = ________________
\]
\[
D = ________________
\]

Calculations:

**Conclusion**

State the width of the slit as determined by your measurements and calculations.

## 2. Double slit interference

**Introduction**

If, instead of a single slit, we have two slits, separated by a distance $d$, a different but related phenomenon occurs as illustrated in *Figure 3* below.

*Figure 3: Double slit interference*
Neglecting the width of each slit, constructive interference occurs when the path of the light ray from one slit is the same as the path from the other slit (this corresponds to point \( P_0 \)), or when the paths differ by a distance equal to an integral number \((1, 2, 3, \ldots)\) of wavelengths of light involved. This behavior is summarized in \( \text{Equation 3} \) below.

\[
d \sin \theta_m = m \lambda \quad m = 0, 1, 2, 3, \ldots
\]

In this equation \( m \) is an integer index that can take on the values \(0, 1, 2, 3, \ldots\) and \(d\) is the distance between the slits.

**Special Note:** Notice that while \( \text{Equation 1} \) describes angles at which destructive interference occurred, \( \text{Equation 3} \) describes angles at which constructive interference occurs.

**Procedure**

- Select the double slit which has a slit width of 0.04 mm and a slit separation of 0.25 mm \((a = 0.04 \text{ mm}, d = 0.25 \text{ mm})\). Notice that this system consists of two slits each having the same width as the single slit you just finished investigating. Thus you should see the same general illumination pattern as with the single slit, but with the addition of bright and dark spots corresponding to the double slit interference maxima and minima. The pattern should look like \( \text{Figure 4} \) below.

\[\text{Figure 4: Double Slit Interference, showing locations of the first three maxima.}\]

- Measure and record the locations of the first five maxima (including the central, \( m = 0 \), maximum), as well as the distance, \( D \), between the slit apparatus and the viewing screen. According to \( \text{Equation 3} \), a graph of \( \sin \theta_m \) versus \( m \) should yield a straight line, the slope of which equals \( \lambda/d \). Make such a graph, calculate its slope and test this prediction. Does your slope equal \( \lambda/d \) to within the accuracy of your measurements (say, within 10%)? If not, check with your instructor for suggestions on improving the accuracy of your measurements.

\[
D = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_
\]

<table>
<thead>
<tr>
<th>( m )</th>
<th>( y ) (cm)</th>
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<tbody>
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<td>0</td>
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<td>1</td>
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3. Multiple slit interference

Introduction

If, instead of two slits, we use a device consisting of many closely-spaced slits, called a diffraction grating, the same general process happens as with two slits. That is, constructive interference occurs at points where the pathlength from any two adjacent slits is either the same (corresponding to the central maximum) or differs by an integral multiple of the wavelength of the light involved.

Several important differences occur, however. First, putting many slits in a relatively small space automatically requires that each slit be quite narrow. This results in the higher-order maxima being located at larger angles relative to the central maximum. Second, since the condition for constructive interference depends on the light from all the illuminated slits, the maxima are all very sharp and well-defined, rather than being somewhat broad and “smeared out” as with the double slit apparatus. However, basically the same thing is going on as with the two-slit case, so Equation 3 still accounts for the location of the points of constructive interference.

Since there are normally many lines in a diffraction grating, it is often the practice for the manufacturer to specify, not the distance between adjacent lines (the $d$ in Equation 3), but rather the number of lines per some specified unit of length, usually millimeters. This number, generally denoted by $N$, is related to $d$ by the equation,

$$d = \frac{1}{N}$$  

(4)

The units of $d$ will be the same as those of the specified length unit given with the value of $N$. Thus if $N = 200$ lines/mm, $d$ will equal 0.005 mm.

Procedure

- Remove the double slit apparatus from the optics bench. At your lab station there should be a diffraction grating (it will look like a 35 mm slide) mounted in a paper clamp. Place the grating in the path of the laser beam with the line on the grating oriented vertically. You will probably have to place the viewing screen much closer

Conclusion

Does your slope equal $\lambda/d$ to within the accuracy of your measurements (10%)?

Calculations: (Attach your graph when you submit your report.)
to the diffraction grating in order for the higher-order maxima to be seen on the screen. In fact you may only be able to see the first-order maxima (in addition to the central maximum, of course). In order to ensure that your measurements are as precise as possible, adjust the location of the viewing screen so that the two first-order maxima just hit the two ends of the scale mounted on the viewing screen. Carefully measure \( D \) and \( y \). Calculate \( \theta_1 \), and use Equation 4 to determine the number of lines per millimeter of your diffraction grating.

\[
y = _________________
\]

\[
D = _________________
\]

Calculations:

Value for \( N \): _________________