ECE 480
Spring/Summer 2007
Second Midterm Test
“Write out and sign the Honor Pledge”

Time: 2 hours and 30 minutes

1) Consider the continuous-time signal
   \[ x_a(t) = e^{-80t} \sin(50\pi t + \pi/3)u_a(t), \]
   where \( t \) is in seconds. This signal is sampled at a rate of 200 samples/sec.
   a) Sketch the signal \( x_a(t) \).
   b) Determine the discrete-time signal \( x(n) \), which is obtained from sampling \( x_a(t) \) at
      the above mentioned rate. Hence sketch \( x(n) \).
   c) Find the z-transform of \( x(n) \) and give the region of convergence.
   d) Does \( x(n) \) have a Fourier transform? Explain.

2) Consider the two discrete-time systems given by following information:
   - **System1**: described by the impulse response \( h(n) = (0.6)^n u(n) \)
   - **System2**: described by the difference equation
     \[ y(n) = 0.8 \{ x(n) - x(n-1) \}. \]

   Suppose that the two systems are connected in series to form a composite system.
   a) Determine the composite transfer function \( H(z) \). Check for stability.
   b) Write the difference equation of the composite system.
   c) Determine the frequency response of the system. Hence find and plot the
      magnitude response. What kind of filter does it represent?
   d) Write an expression for the output if the input is given as follows:
      \[ x(n) = \begin{cases} 1, & 0 \leq n \leq 4 \\ 0, & \text{otherwise} \end{cases} \]

   **Hint**: Express \( x(n) \) in terms of other elementary signals. Hence find the output of the
   components of \( x(n) \).
3) Consider the causal sequence $x(n)$ with the $z$-transform given as follows:

$$X(z) = \frac{z^{-1} - 1.2z^{-2} + z^{-3}}{1 - 1.3z^{-1} + 1.04z^{-2}}$$

a) Determine $x(n)$ for $n = 0, 1, 2, 3$ and $4$. Sketch $x(n)$ for $n = 0, 1, 2, 3$ and $4$.

b) If the signal $y(n)$ has the $z$-transform given by $Y(z) = z^2 \frac{dX(0.5z)}{dz}$, find $y(n)$ for $n = 0, 1, 2$. 