Team:____________________

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Acceleration

Part I. Uniformly Accelerated Motion: *Kinematics & Geometry*

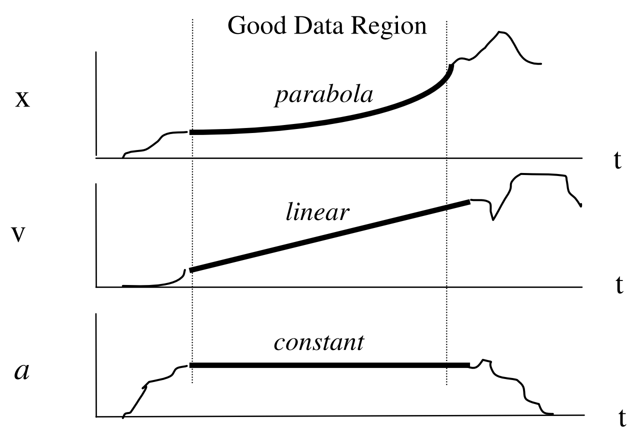
Acceleration is the rate of change of velocity with respect to time: $\boldsymbol{a} \equiv d\mathbf{v}/dt$. In this experiment, you will study a very important class of motion called uniformly-accelerated motion. Uniform acceleration means that the acceleration is constant – independent of time – and thus the velocity changes at a constant rate. The motion of an object (near the earth's surface) due to gravity is the classic example of uniformly accelerated motion. If you drop any object, then its velocity will increase by the same amount (9.8 m/s) during each one-second interval of time.

Galileo figured out the physics of uniformly-accelerated motion by studying the motion of a bronze ball rolling down a wooden ramp. You will study the motion of a glider coasting down a tilted air track. You will discover the deep connection between *kinematic* concepts (position, velocity, acceleration) and *geometric* concepts (curvature, slope, area).

A. The Big Four: t , x , v , *a*

The subject of *kinematics* is concerned with the description of how matter moves through *space* and *time*. Time t , position x , velocity v , and acceleration a are the basic descriptors of any kind of motion of a particle moving in one spatial dimension. They are the "stars of the kinema". The variables describing space (x) and time (t) are the fundamental kinematic entities. The other two (v and a) are derived from these spatial and temporal properties via the relations $v \equiv dx/dt$ and $a \equiv dv/dt$.

Let's measure how x, v, and a of your glider depend on t. First make sure that the track is level. The acceleration of the glider on a horizontal air track is constant, but its value ($a = 0$) is not very interesting. In order to have $a \neq 0$, you must tilt the track. Place two wooden blocks under the leg of the track near the end where the motion sensor is located. Release the glider at the top of the track and record its motion using the motion sensor. [Click on *Logger Pro* and open file *Changing Velocity 2*]. The graph window displays x, v, and a as a function of time t. Your graphs should have the following overall appearance:



Change the x scale of your graph to go from 0 to 2 m. Change the v scale to go from 0 to 1 m/s. Change the a scale to go from 0 to 0.5 m/s^2. Identify the good data region of your graphs where the *acceleration is constant*. Change the time (t) scale so that the good data region fills most of the whole graph window – like the graphs on the previous page. Your x vs t graph should be a smooth curve (*parabola*), your v vs t graph should be a sloping line (*linear*), and your a vs t graph should be a flat line (*constant*).

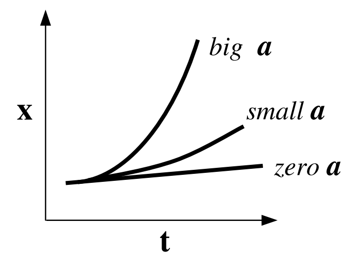
Note: In the "bad data region", which you want to eliminate from your graphs, the acceleration is changing because the glider is experiencing forces other than gravity, such as your hand pushing the glider or the glider hitting the bumper.

PRINT the whole screen so that your x , v , a graphs all appear on one page. Remember to write a short title. If you are not sure that your graphs show the good data region, then see your instructor.

B. Acceleration = "Curvature" of x(t)

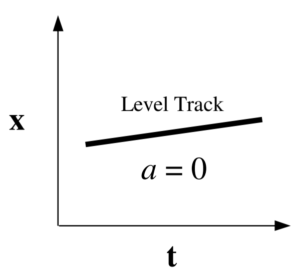
Look at your x(t) graph and note: The worldline of your glider is *curved* ! Recall that in the *Constant Velocity* lab, all graphs were *straight*. *Changing Velocity* is synonymous with a *Curved Worldline*:

Acceleration		Changing Velocity		Curving Worldline
a	=	dv/dt	=	d^2x/dt^2 .

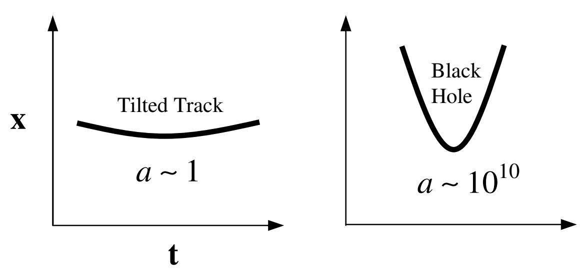


The amount of "bending" in a curve – the deviation from straightness – is measured by how much the slope changes. Acceleration – the rate of change in the slope of x(t) – measures the curvature of spacetime.

"Flat Spacetime"



Curved x(t) ⇔ "Warped Spacetime"



The Importance of "Curvature" in Theoretical Physics

The *mass* of the earth is the ultimate cause of the *curved worldline* of your glider. Remove the earth and the worldline would become straight. Two Hundred and Fifty years after Newton, Einstein formulated his celebrated *"Field Equations"* of General Relativity which state the precise mathematical relationship between the *amount of mass* (the source of gravity) and the *curvature of spacetime*. Force causes x(t) to curve. Mass causes spacetime to warp.

Measuring Curvature

Here you will measure the curvature of your glider's worldline x(t). Select the good-data region of your x(t) graph. Click on the *Curve-Fit Icon* [f(x)=?] and perform a *"Quadratic Fit"* to find the best-fit curve (parabola) through the x-t data points. Recall that the equation of a kinematic parabola in physics is $x(t) = \frac{1}{2} a t^2 + v_0 t + x_0$. The computer will give the equation in purely mathematical language: "y as a function of x". Write this equation in physics language: "x as a function of t":

Worldline of Glider: x(t) = ____________________________________ .

Based on this best-fit worldline, what is the acceleration *a* of your glider? Note: you can find *a* from x(t) two different ways: (1) *Algebraic Method*: note that the coefficient of t^2 in x(t) is $a/2$.
(2) *Calculus Method*: take the second derivative of x(t), i.e. $a = d^2x/dt^2$.

"Curvature" of x(t) (Acceleration of glider): a = ________________ m/s^2 .

C. Acceleration = Slope of v. Displacement = Area under v.

In the previous section, you found *a* from the x(t) graph via the relation $a = d^2x/dt^2$ (*curvature*).
In this section, you will find *a* from the v(t) graph via the relation $a = dv/dt$ (*slope*).
You will also find Δx from the v(t) graph via the relation $\Delta x = \int v dt$ (*area*).

Pick two points on the v(t) line within the good-data region that are not too close to each other. Find the values of t and v at these points using the *Examine Icon* [x=?] or from the data table. Also find the position x of the glider at these same two times.

t_1 = ____________ *s* v_1 = ____________ *m/s* x_1 = ____________ *m* .

t_2 = ____________ *s* v_2 = ____________ *m/s* x_2 = ____________ *m* .

PRINT your v(t) graph. Label the points 1 and 2 with your pen. Write the coordinate values (t_1 , v_1) and (t_2 , v_2) next to each point. Calculate the following two <u>geometric</u> properties of the v(t) graph:

1. *Slope* of the line.

   [rise over run]

2. *Area* under the line between t_1 and t_2 .

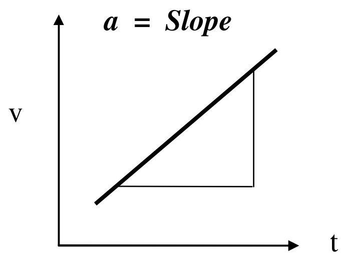
   [area of rectangle + area of triangle]

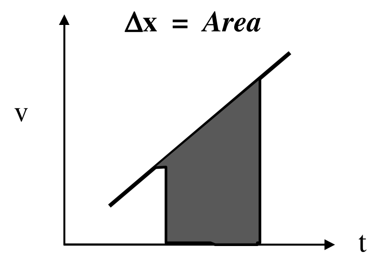
Show your calculations (rise-over-run , base-times-height, etc.) *directly on your printed graph*. Report your slope and area results here:

Slope of v(t) *line* = ____________ *(m/s) / s.*   *Area under* v(t) *line* = ____________ *(m/s) • s .*

Mathematical Facts:
1. The ratio dv/dt is the rise (dv) over the run (dt) of the v(t) line.
2. The product vdt is the area of the rectangle of base dt and height v.

Physical Consequences:
1. a = dv/dt ⇒ *Acceleration* a = *Slope of* v(t) *graph* .
2. dx = vdt ⇒ *Displacement* Δx = *Area under* v(t) *graph* .





You already found a from the curvature of x(t). Write this value of a again in the space below. From your measured values of x_1 and x_2 (listed above), you can find the displacement of the glider: $\Delta x = x_2 - x_1$, i.e. the distance moved by the glider during the time interval from t_1 to t_2 .

a = ____________ m/s^2 .   Δx = ____________ *m* .

Compare this value of a with your value of *"Slope of* v(t) *line"*. Compare this value of Δx with your value of *"Area under* v(t) *line"*.

% diff between a and *slope* = ______ %.   % diff between *Δx* and *area* = ______ %.

Physics & Calculus

The problem of finding *slopes* and *areas* is the essence of the whole subject of Calculus. Newton invented *Calculus* to understand *Motion*. In Calculus, "*finding slopes* (accelerations)" and "*finding areas* (displacements)" are inverse operations called "*differentiation*" and "*integration*", respectively. In the language of mathematics, $a = dv/dt$ and $\Delta x = \int v dt$.

Part II. The $x \propto t^2$ Theorem

The central kinematic equation in the works of Galileo and Newton is

$$x = \frac{1}{2} a t^2 .$$

This equation says that the position *x* of any object that starts from rest and undergoes constant acceleration *a* is a quadratic function of the time *t*. You will use this equation in many of the future labs.

Theoretical Exercise. In general, the worldline x(t) for uniformly-accelerated motion is $x(t) = \frac{1}{2} a t^2 + v_0 t + x_0$. Calculate v = dx/dt. Solve the resulting equation for *a* in terms of v, v_0 , and t. Interpret your result.

Experimental Test of the *Squared* Relation $x \propto t^2$

We can assume the acceleration of the glider on an inclined air track is constant (neglecting air friction). Hence the distance x travelled by the glider along the track is proportional to the *square* of the time elapsed (after starting from rest). This means that if you *double* the time, $t \rightarrow 2t$, then the distance will *quadruple*, $x \rightarrow 4x$. More specifically, if it takes time t_1 to move distance x_1 and time t_2 to move distance x_2 , then the proportionality $x \propto t^2$ implies the following equality of ratios: $x_2/x_1 = (t_2/t_1)^2$. This ratio relation says if $t_2 = 2t_1$, then $x_2 = 4x_1$.

Start with the tilted track with two blocks under the end of the track. Use a *stopwatch* – not the motion sensor – to measure the time it takes the glider, *starting from rest*, to move a distance of 25 *cm* down the track. Experimental Techniques: (1) The time measurement will be most accurate if you start the glider at a point that is 25 *cm* away from the rubber band at the lower end of the track. Seeing and hearing the glider hit the rubber band tells you the precise moment to stop the stopwatch. (2) The same person should release the glider and time the motion in order to minimize "reaction time error".

Repeat three more times and find an average time. Next measure the time it takes, starting from rest, to move a distance of 100 *cm*.

					Average Time
t (x = 25 *cm*)					(s)
t (x=100 *cm*)					(s)

Are your experimental results consistent with the theoretical relation $x \sim t^2$? Explain carefully by *constructing ratios*. Hint: Calculate x_2/x_1 and $(t_2/t_1)^2$.

Part III. The Galileo-Einstein Principle

One of the deepest facts of Nature is this:

> *The acceleration of an object due to gravity does not depend on the size, shape, composition, or mass of the object.*

In the absence of friction, all bodies fall at the same rate! Einstein's *General Theory of Relativity* – which says that gravity is due to spacetime curvature – was motivated by Galileo's observation that free fall is independent of mass.

Experimental Test of the Universality of g

Strictly speaking, *free fall* refers to the vertical motion of a body that is *free* of all forces except the force of gravity. A body moving on a friction-free inclined track is falling freely along the direction of the track. It is *non-vertical free fall motion*. The track simply changes the direction of the fall from vertical to "diagonal". This diagonal free fall is a slowed-down and thus easier-to-measure version of the vertical free fall. The acceleration along the track is the diagonal component (g sinθ) of the vertical acceleration (g).

Use two blocks to incline the track. Use the motion sensor to record the motion of the glider as it falls freely down the track. Remember to carefully select the *good data* (constant *a*) region of the graph before you analyze the data. Find the acceleration of the glider by averaging the *a* versus t data: click on the statistics icon [*STATS*]. Report your results in the table below. For example, if the statistical analysis of the acceleration data gives the average value 0.347 m/s^2 and the standard deviation 0.021 m/s^2, then you would report your measured value of acceleration to be 35 ± 2 cm/s^2. The *range* of this *a* is $33 \rightarrow 37$ cm/s^2.

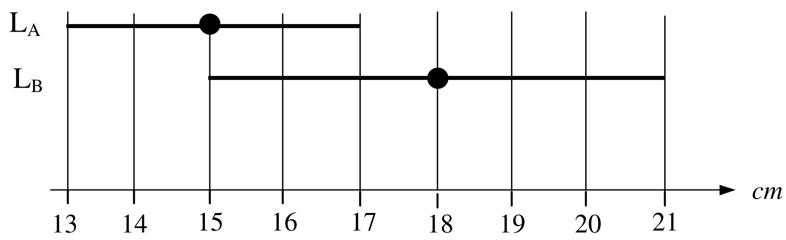
Add two weights or "metal donuts" (one on each side of the glider) and measure the acceleration. Add four weights (two on each side) and measure the acceleration.

Mass	$a \pm$ *uncertainty* (cm/s^2)	*Range* of *a* (cm/s^2)
0 added weights	±	→
2 added weights	±	→
4 added weights	±	→

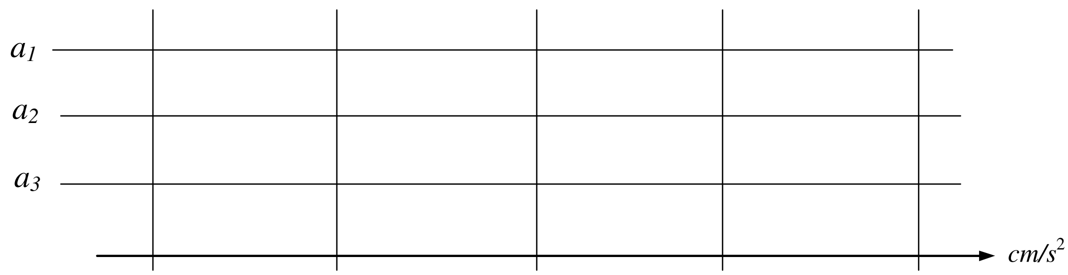
Your values of *a* may look "close", but can you conclude that they are "equal" ? The word "close" is not part of the language of science.

<u>When are two experimental values *"Equal"* ?</u>

To answer this question, the role of uncertainty is vital. A measured value such as 15 ± 2 is really a *range* of numbers $13 \rightarrow 17$. *Two experimental values are equal if and only if their ranges overlap*. Suppose you are given two rods (A and B) and measure their lengths to be $L_A = 15 \pm 2$ *cm* and $L_B = 18 \pm 3$ *cm*. Since the two ranges overlap, $13 \rightarrow 17$ and $15 \rightarrow 21$, you can conclude that these two rods are equal in length. A *range diagram* provides an excellent visual display of the experimental values of measured quantities. The following range diagram for L_A and L_B clearly exhibits the amount of overlap:



Plot your three measured values of *a* – the acceleration of gravity (along the track) – on the following range diagram:



Now you can rigorously answer the important question: Do your measured values of *a* provide an experimental proof of the deep principle that the *acceleration of gravity is independent of mass*? Explain.

Einstein , Curved Space , Black Holes , Warp Drive

In a gravitational field, *all* bodies fall with the *same* acceleration. Einstein used this law as the basis for his *general theory of relativity*. All bodies fall in the same way because they are merely "coasting along" the same "downhill" contours of the curved space that they happen to occupy. Einstein's field equations tell you precisely how to calculate the curvature of four-dimensional space-time. *Gravity is not a force – it is the shape of space*. The idea that "gravity is curvature" is the basis for warped space, bending light, gravity waves, black holes, and wormholes.

In essence, your experimental proof that *"g is independent of m"* is a proof of the existence of black holes and gravity waves! When warp drive is invented, you will appreciate that it is a consequence of the universality of g.

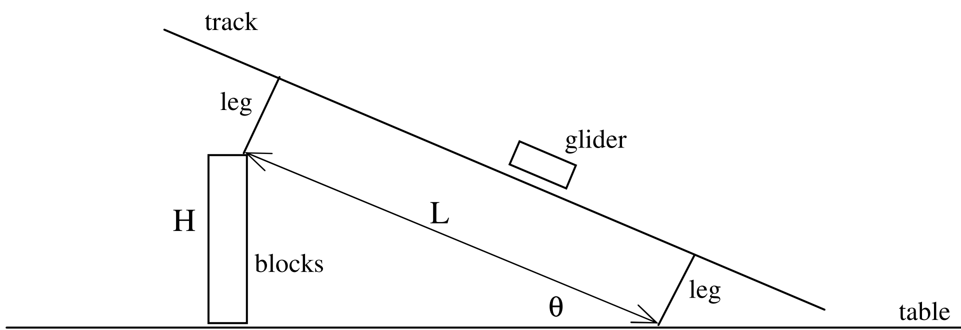
<u>Comments</u>:

Part IV. Designing a Diluted-Gravity System

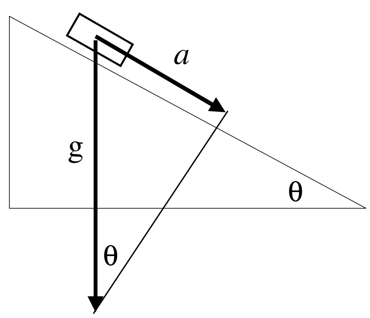
In vertical free fall, an object released from rest moves 60 *m* in 3.5 s. You need to slow this motion – dilute gravity – so that the object only moves 1.5 *m* in 3.5 *s*. Your goal is to find how much the track needs to be tilted to achieve this slowed-down motion. First work out the theory and then perform the experiment.

The Theory

Architecture Diagram. H = height of blocks. L = distance between track legs. θ = angle of incline.



Acceleration Diagram. g = *full strength gravity*. *a* = *diluted gravity*.



Note: The component of g along the track direction is gsinθ. As an example, for $\theta = 30^o$, the "dilution factor" is sinθ = ½ and thus the acceleration is $a = g/2$. For $\theta = 72^o$, $a = 0.95g$.

Two Step Solution

1. Calculate the acceleration that the glider must have in order to satisfy the *Design Specs*: "glider released from rest and moves 1.5 *m* in 3.5 *s*".

a = ______________ m/s^2 .

2. Calculate the height H of the blocks that is necessary to achieve this amount of acceleration. *Hint*: H is related to sinθ and L (see architecture diagram), but sinθ is related to *a* and g (see acceleration diagram). *Caution*: L ≠ 1.5 m.

H = _______________ *cm* .

The Experiment

1. Again, make sure that the track is level. Raise the end of the track by the height H predicted above. To achieve this value of H (to within tenths of a centimeter), you will most likely need to stack thin square metal plates on top of the wooden block(s).

2. Release the glider from rest. Use a stopwatch to measure the time t it takes the glider to move 1.5 *m* along the track. To achieve greater accuracy in timing, measure the 1.5 m distance from the elastic cord (bumper) that the glider hits at the *lower* end of the track. Seeing/hearing the glider hit the cord provides a well-defined signal for you to stop the watch.

Repeat this measurement five times. List your five values of t below and compute the average time and the uncertainty in the time. Estimate the *uncertainty* (deviation from average) from the "half-width spread" in your five values of time: *Uncertainty* = $(t_{max} - t_{min})/2$.

t (s)					

t = ____________ ± ____________ s .

3. Compare this measured value of time with the design goal of t = 3.5 *s*. What is the percent difference? Clearly show whether or not the *theoretical* time of 3.5 seconds falls within your *experimental range* ? What source(s) of error could account for any discrepancy?